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Investigation of critical sustainability decisions in product recycling and remanufacturing

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Investigation of critical sustainability decisions

in product recycling and remanufacturing

by

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A dissertation submitted to the graduate faculty

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ACKNOWLEDGEMENT

ABSTRACT

In recent years, there have been substantial interests in sustainable processes for the end-of-life products to conserve natural resource and reduce landfill wastes. Among these processes, recycling and remanufacturing provides opportunities for practitioners from various industries and government regulators to mitigate the environmental impacts while maintaining economic viability. Meanwhile, there are substantial critical decisions for the decision/policy makers to reap to the economic and environmental benefits from the recycling and remanufacturing processes. Therefore, in this study, I investigate the product weight reduction and used-product collection decisions in the context of closed loop supply chains with product recycling. Furthermore, from a real options perspective, I investigate the optimal timing of remanufacturing with the remanufacturing cost following a stochastic process, and the optimal timing of remanufacturing and replacement with the maintenance costs following stochastic processes. Also, in this study, I investigate the influences of the governmental economic instruments on the recycling and remanufacturing decisions, and examine the environmental viability of these sustainable incentives. A series of managerial insights and policy implications are obtained from analytical and numerical analyses.

CHAPTER 1

INTRODUCTION

In recent years, there have been substantial interests in the sustainable processes for the end-of-life products to reduce resource consumption and landfill wastes. Among these processes, recycling and remanufacturing provides opportunities for firms from a variety of industries to mitigate the environmental impacts while maintaining economic viability. Toner cartridge recycling is a 3 billion dollar industry [1] and 90% (in weight) of the recycled materials are utilized in the new toner cartridges [2]. Also, Panasonic diverted thousands of tons of cathode ray tubes (CRT's) from landfill through recycling process, and utilized the recycled glass, metal, and plastics in the production of new CRT's [3]. within the U.S., the remanufacturing industry involves more than 73,000 firms and 350,000 people with annual revenue of $53 billion [4]. Overall, remanufacturing programs can save companies 40%–65% of the manufacturing cost [1]. Xerox's remanufacturing program saved $200 million in material cost in less than five years [5].

Therefore, from a decision/policy maker's perspective, there are substantial opportunities to reap the economic and environmental benefits of the remanufacturing and recycling processes, such as closed loop supply chain management, sustainable product design, used-product acquisition strategy, remanufacturing timing, as well as governmental subsidies and fees.

With this knowledge, in this study, we aim to understand the economically rational decisions on (i) the product weight reduction and used-product collection efforts in the context of closed loop supply chains with recycling; (ii) the product remanufacturing decision with uncertain re-

manufacturing cost; as well as (iii) the product remanufacturing and replacement decisions with uncertain O&M cost (i.e., operations and maintenance cost). Specifically,

Toward the research objective (i), we formulate three closely related CLSC models consisting of a manufacturer who also recycles used products and a collector of the used products. The first CLSC is a centralized model with the vertical integration of the manufacturer and the collector. The second CLSC is a decentralized model with the manufacturer as the leader and the collector as the follower in a Stackelberg game. The third CLSC is a decentralized model with government subsidy and fee. In addition, we formulate a non-recycling model as a benchmark for comparison.

Under this framework, the analytic results show that (a) conditions (e.g., the increase (decrease) of the marginal recycling benefit or the collecting subsidy) leading to a higher (lower) level of the collection rate may also result in a higher (lower) level of the product weight, and vice versa. Hence, decision makers may not be able to pursue an improvement of both factors simultaneously, and it is advised to consider the negative impact on the other factor before any efforts on improving one factor. As a consequence, (b) the increase (decrease) of the marginal recycling benefit or the collecting subsidy (for a unit weight of used-product collected) may result in a higher level of the landfill quantity when the marginal recycling benefit or the collecting subsidy is sufficiently high (lower). In this sense, efforts on improving one factor should not be supported unconditionally, as it may make the environment worse off. (c) Under the revenue-neutrality framework (i.e., the amount of subsidies disbursed balances the amount of fees collected), the landfill quantity in the centralized model can be achieved or even further reduced in the decentralized model by choosing a proper value of subsidy. However, inappropri-

ate value of subsidy may also result in inferior environmental performance (i.e., more landfill quantity) relative to the non-subsidy case. Given these findings, the pricing of subsidy/fee becomes critical to achieve high environmental efficiency and avoid unintended negative environmental consequences.

Toward the research objective (ii), we consider a firm that leases a single product to a service provider, and the firm is also responsible for the maintenance of the product. For instance, Xerox leases photocopiers to Staples which provides copying services to customers. Throughout a product's lifecycle, as the maintenance cost increases (due to component-deterioration), at any time point, the firm has an option to terminate the life of the product and remanufacture it. During the remanufacturing process, depending on the physical condition, components are either reused or replaced and disposed. After the remanufacturing process, the product is restored to a like-new condition and re-leased to the service provider. We note that the flexibility on remanufacturing described above lends itself to a real options approach. That is, the firm has the right, but not the obligation, to make changes to its business project under uncertainty. Moreover, in the extended scenario, government participation is incorporated into our model in the form of remanufacturing subsidy and disposal fee.

The critical findings of research topic (ii) include: (a) The uncertainty of remanufacturing cost is the driver for decision makers to prolong the product life and defer the exercise of remanufacturing option because the uncertainty increases the value of holding the flexibility; (b) Increasing the remanufacturing subsidy (disposal fee) incentivizes decision makers to exercise the remanufacturing option earlier (later) because the subsidy reduces (increases) the cost of exercising the option now. That is, the value of waiting is diminished; (c) An increase of the remanu-

facturing subsidy (disposal fee) may result in more industrial wastes because it entails a shorter product life (lower reuse fraction of materials); (d) As the remanufacturing cost becomes more uncertain, the policy maker is advised to increase the remanufacturing subsidy or decrease the disposal fee because these adjustments balance the higher level of landfill disposal stemmed from the higher uncertainty. We hope that these findings, as the first research findings on the remanufacturing decisions via a real options approach will stimulate relevant discussions among industrial practitioners, governmental regulators, environmental groups, as well as academics.

Toward the research objective (iii), we consider a firm that leases a product to a service provider, and the firm is assumed to be responsible for the O&M costs of the product. The O&M costs for the durable parts and the non-durable parts are assumed to follow uncorrelated geometric Brownian motion (GBM) processes. Throughout the product's life-cycle, the firm has two options for the product: remanufacturing option and replacement option. With these options, the firm has the right, but not the obligation, to remanufacture or replace the product at any point. In particular, the exercise of the remanufacturing option triggers the ultimate maintenance for the product by replacing the non-durable parts with new ones. The replacement option triggers the replacement of the product as a whole.

The analytical and numerical findings in (iii) include that (a) as the O&M costs become more (less) volatile, it is beneficial to exercise the remanufacturing and the replacement options later (earlier); (b) a higher (lower) remanufacturing cost results in a shorter (longer) expected product life because it becomes less attractive for the firm to conduct ultimate maintenance for the product through remanufacturing; (c) an increase of the remanufacturing subsidy may result in

more landfill disposal per unit time, and the government policy maker is advised to carefully examine the industrial conditions before any implementation of a new sustainable subsidy or adjustment of the existing sustainable subsidy. We hope that the questions raised and addressed in this study will stimulate relevant discussions among industrial practitioners, governmental regulators, and academics.

The rest of the article is organized as follows. Chapter 2 provides a comprehensive literature review relevant to this research. In Chapter 3, we formulate and analyze a series of closed loop supply chain models for the product weight reduction and the collection rate decisions with product recycling. In Chapter 4, we formulate and analyze the models for the product remanufacturing decision from a real options perspective with the remanufacturing cost following a GBM process. Similarly, we utilized the real options approach to investigate the remanufacturing and replacement decisions when the maintenance costs are assumed to following the GBM processes in Chapter 5. Finally, the conclusions and future works are presented in Chapter 6.

CHAPTER 2

LITERATURE REVIEW

As this research is associated with several streams of research topics, we will review the relevant articles of each topic as follows.

A growing literature in CLSC and reverse logistics addresses management issues such as channel design [11][13], technical investments [70], and pricing strategy [20]. In these papers, the environmental performance is measured by a single factor such as return rate [11][13], remanufacturability [70], and collection quantity [20]. Geyer et al. [5] demonstrated the significance of coordinating the investments in product durability and collection rate. However, the study is based on numerical analysis, and analytic results are not observable. In this paper, by considering variable product weight and collection rate, we examine the interaction between two factors and their environmental consequences.

There are numerous articles relevant to product take-back and recycling focusing on tactical and operational issues such as inventory control [71], shop floor control [72], logistics network design [73], and material resource planning [74]. We refer readers to [4] a comprehensive literature review. Recently, there has been a shift in emphasis towards the economics of product reuse and recycle. Guide et al. [12] demonstrated that the product acquisition and cost saving are the critical drivers of recycling profitability. Debo et al. [75] investigated the impact of consumer profiles and technology investment on the recycling profitability. Our study investigates

the economic relationship between the product weight and the collection rate in the context of CLSC's.

We now proceed to review the relevant articles on product weight. Huang et al. [6] claimed that light weight have been taken as the design principles of electronic products. Kleiner et al. [76] pointed out the significance of lightweight design in engineering, transportation, and architecture industries, and demonstrated the feasibility of reducing product weight via utilizing lighter materials and/or reducing the product size. Atasu et al. [12] pointed out that, since WEEE targets are unilaterally weight-based, most producers do not have an incentive to invest in environmental design. Hence, our study examines the interaction between investment in product weight reduction and used product taking back activities. Furthermore, Atasu et al. [12] also claimed that the cost to the environment depends on the product's physical properties such as weight. Align with this claim, landfill quantity (i.e., total weight of products sent to landfill) is utilized to in our models to measure the environmental impact of the CLSC's.

The use of taxes and subsidies as a tool to improve social welfare has been widely studied [77]. Carraro and Topa [78] analyzed the impact of environmental regulation in the form of taxation on the innovation activity of firms. Palmer and Walls [38] utilized a tax/subsidy combination in which producers of intermediate goods pay a per-pound tax and collectors of used products receive a per-pound subsidy from the tax. Aligning with Palmer and Walls, under the revenue-neutrality framework, we implement a subsidy/fee system which charges the manufacturer a weight-based production fee, and subsidies the collector for the weight of products collected. We note that Mrozek [39] argued that, to achieve the maximum efficiency, deposit/refund systems must be implemented in a way that the revenue is neutral.

Furthermore, in recent years, there have been increasing interests in examining the uncertainties in decision making process of product remanufacturing. Denizel et al. [58] investigated a hybrid production planning problem where the inputs (i.e., used products) are subject to different and uncertain quality levels. In [74], it was demonstrated that the remanufacturing cost is approximately linear to the quantified quality deterioration. Robotics et al. [79] also considered a firm that remanufactures products and makes investment decisions on increasing the reusability of its products. Assuming a normal distribution of the remanufacturing cost, they claimed that uncertainty of the remanufacturing cost may not hinder this investment, depending on the inspection capabilities and technologies. Similarly, by assuming a uniform distribution of product quality, Galbreth and Blackburn [57] investigated the optimal acquisition and sorting policies. Siddiqui and Marnay [80] investigated the investment decision on a distributed generation unit fuelled by natural gas for a microgrid. By modeling the natural gas generation cost as stochastic process, they found the cost threshold that triggers the investment. Unlike [57][79][80], this paper studies the dynamic feature of the remanufacturing cost by assuming it follows a stochastic process. Also, we demonstrate that the uncertainty of the product quality actually motivates decision makers to defer the remanufacturing. Readers may refer to Guide [4] for a thorough review on the uncertainties in the product remanufacturing process.

The real options analysis, originated from finance, has been extended to the decision making processes with uncertainties in expansion, replacement, and abandonment. Adkins and Paxson [81] investigated the switching option between two energy sources by assuming that feedstock prices follow GBM. Wickart and Madlener [82] investigated the optimal technology investment timing for power plants with both fuel price and electricity price following GBM. These articles

differ from our approach in the sense that the prices are modeled as stochastic, not the cost components. On the other hand, Ye [83] modeled the maintenance cost as a GBM process and utilized the optimal stopping method to solve the equipment replacement problem. Zambujal-Oliveira and Duque [53] extended the replacement problem by considering maintenance cost and equipment salvage value as GBM processes. More recently, Min et al. [84] investigated the entry and the exit of the renewable power producers by modeling the operating and maintenance cost as GBM. Adkins and Paxson [81] studied the optimal replacement and abandonment decisions for real assets. The exit decision in [84], the replacement decision in [81], and the remanufacturing decision in this paper can all be considered as optimal-stopping problems. However, this paper, to our knowledge, is the first study to model remanufacturing cost as a stochastic process, and investigate the optimal remanufacturing timing from a real options perspective. The remanufacturing cost uncertainty stems from the uncertainties in end-of-life-product quality and component replacement cost.

Furthermore, in recent years, there has been increasing interest in utilizing the real options approach in engineering disciplines. Mikaelian et al. [85] developed a qualitative study for a holistic consideration of real options in enterprise architecture through mechanisms and types. The benefit of this new development was demonstrated over traditional real options analysis in identifying a broader spectrum for uncertainty management. Cardin et al. [86] proposed a five-phase taxonomy of procedures to support the design and management of engineering systems with uncertainty and flexibility. In this paper, decision makers recognize the uncertainty in remanufacturing cost, and have flexibility on the timing of remanufacturing. Readers may refer to [87][88] for more comprehensive reviews on real options in an engineering context.

CHAPTER 3

PRODUCT WEIGHT AND COLLECTION RATE DECISIONS IN CLOSED LOOP

SUPPLY CHAINS WITH PRODUCT RECYCLING

3.1   Background and Objectives

In recent years, there have been significant interests in sustainable processing of end-of-life products such as recycling to reduce the consumption of raw materials and the landfill disposal. A significant part of them is driven by the increasing economic motivations and environmental concerns in communities in general, and the manufacturers, collectors, as well as government in particular.

A critical factor relevant to the environmental sustainability is the product weight. In recent years, lightweight have been taken as the design principle of electronic products [6]. From an environmental perspective, reducing the product weight not only brings down the consumption of natural resources, but also reduces the waste disposed at landfills.

From an economic perspective, a lower level of the product weight implies less materials cost and more energy savings. Xerox claimed substantial energy savings and diverted a significant amount of materials from the landfill via lightweight design of toner cartridges [7]. The product weight could be reduced via utilizing lighter materials [8] and/or reducing the product size [9][10]. By shrinking curved zoom lenses, Konica Minolta reduces the weight of single use camera by one third in the last decade [9].

Another critical factor for the environmental sustainability is the collection rate, which is the fraction of the sold products that are collected, and measures the collection efficiency in the reverse channel. End-of-life products are taken back to manufacturer [11], and go through the value-retrieval process such as remanufacturing or recycling. Hence, the acquisition of used products is a key driver for the success of product reuse and recycle [12]. A higher level of the collection rate results in both less consumption of new raw materials and less waste in landfills, and can be achieved, for example, by investing in advertising/educational campaigns to enhance the public awareness of the take-back programs [13].

Given the significance of both environmental factors, it is highly desirable to understand how efforts on improving one factor impact the other factor and the total environmental consequence. Even though numerous environmental targets set by governments are weight-based [13], only few analytical studies explain and enhance our understanding such an impact from an economically rational perspective.

As a first step toward this goal of deeper analytical understanding, in this paper, we examine the variable product weight and collection rate, as well as their environmental consequence of the landfill quantity via a series of straightforward closed-loop supply chains (CLSC's).

Essentially, our attempts to answer the following research questions

(1) Under which condition, product recycling is economically viable?

(2) How the marginal recycling benefit (with respect to the product weight) influences the product weight, the collection rate, and the landfill quantity (i.e., the total weight of products disposed at landfill)?

(3) How supply chain centralization/decentralization influence the product weight, the collec-

tion rate, and the landfill quantity?

(4) How government subsidy/fee influence the product weight, the collection rate, and the landfill quantity?

(5) Under which condition, subsidizing the collecting activities and taxing the production activities are environmentally viable? How the high level of environmental efficiency (in terms of landfill quantity) in the centralized CLSC model can be achieved or outperformed in the decentralized CLSC model by setting appropriate subsidy and fee?

Specifically, to answer these questions, we formulate three closely related CLSC models consisting of a manufacturer who also recycles used products and a collector of the used products. The first CLSC is a centralized model with the vertical integration of the manufacturer and the collector. The second CLSC is a decentralized model with the manufacturer as the leader and the collector as the follower in a Stackelberg game. The third CLSC is a decentralized model with government subsidy and fee. In addition, we formulate a non-recycling model as a benchmark for comparison.

Under this framework, the analytic results show that (i) conditions (e.g., the increase (decrease) of the marginal recycling benefit or the collecting subsidy) leading to a higher (lower) level of the collection rate may also result in a higher (lower) level of the product weight, and vice versa. Hence, decision makers may not be able to pursue an improvement of both factors simultaneously, and it is advised to consider the negative impact on the other factor before any efforts on improving one factor. As a consequence, (ii) the increase (decrease) of the marginal recycling benefit or the collecting subsidy (for a unit weight of used-product collected) may result in a higher level of the landfill quantity when the marginal recycling benefit or the collect-

ing subsidy is sufficiently high (lower). In this sense, efforts on improving one factor should not be supported unconditionally, as it may make the environment worse off. (iii) Under the revenue-neutrality framework (i.e., the amount of subsidies disbursed balances the amount of fees collected), the landfill quantity in the centralized model can be achieved or even further reduced in the decentralized model by choosing a proper value of subsidy. However, inappropriate value of subsidy may also result in inferior environmental performance (i.e., more landfill quantity) relative to the non-subsidy case. Given these findings, the pricing of subsidy/fee becomes critical to achieve high environmental efficiency and avoid unintended negative environmental consequences.

3.2   Closed Loop Supply Chain Models

For a single product, we consider a supply chain consisting of a manufacturer who sells directly to his/her customers and a collector who accepts used products from the customers and returns the collected used products to the manufacturer for a fee. The manufacturer in turn recycles basic materials from the returned products in manufacturing his/her product.

We note that, for the product in this paper, the fraction of each basic material in weight is fixed (e.g., 35% glass, 25% metal, 35% polymer), and does not vary across units of the product. We also note that the portion of the non-recyclable materials in this product is assumed to be negligible while any used product, that is not collected, is disposed at a landfill. We further note that a third-party collector, who is only engaged in the collection activities of used products and serves as an intermediary between the manufacturer and his/her customers, is often observed in various markets [11].

More quantitatively, in the forward channel, the manufacturer sells $D(p)$ units of the product to customers where p ($/unit) denotes the selling price. In the reverse channel, τ fraction of the sold products is returned to the collector after use, and is in turn supplied to the manufacturer. The τ fraction is the collection rate while the manufacturer's fee to the collector is denoted by b ($/lb).
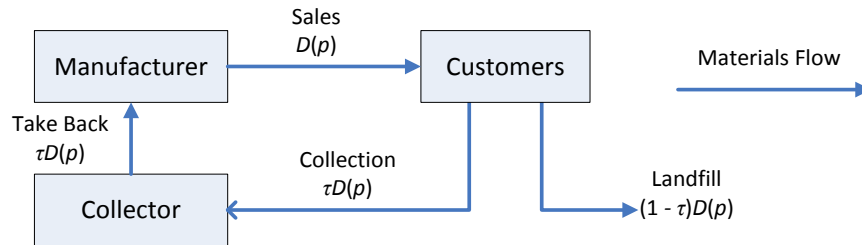
Figure 1. Manufacturer-Collector Closed Loop Supply Chain

Figure 1 depicts such a closed-loop supply chain, and we note that the aforementioned CRT's is a representative product for such a chain as used CRT's are recycled into basic materials such as glass, metal, and plastics [14]. In what follows, we explain the key assumptions of this paper.

Assumption 3.1   *The planning horizon is a static single period.*

All the decision variables are decided in a single period (see e.g., [11]). This assumption enables us to concentrate on the fundamental relationships of product weight vs. collection rate, centralization vs. decentralization, and without government intervention vs. with government intervention without being distracted by dynamic ramifications, which are beyond the scope of this paper.

Assumption 3.2   *The investment cost to reduce the product materials weight from w_0 to w is $C_w(w) = S\ln(w_0/w)$ ($) where w is the product weight while $s > 0$ and $0 < w < w_0$.*

Recently, we observe that lighter weight has become a major business initiative in electronic products [6]. For example, Konica and Fujifilm have reduced the weight of single use camera by 40% in the last decade [9][10]. Meanwhile, the emphasis on lighter weight leads to an increase in equipment cost [15][16] as more precise and delicate tools are required to produce the lighter and smaller-size products.

Under these circumstances, we make a simplifying assumption that any product composition changes due to weight reduction are negligible. Hence, we explained before, the fraction of each basic material in weight is still fixed. A representative example is the case of Xerox, which was able to reduce the weight of toner cartridges without hard composition change by precisely adjusting the parison extrusion rate, thickness, and distribution [17].

As for the functional form, we note that a logarithmic function is utilized to characterize the diminishing returns (from a higher to a lower level of product weight) to investment. Maly [18] utilized the same functional form to depict the rapidly growing design cost of integrated circuits with the decrease of circuit size. Porteus [19] also used similar functional form to investigate the investing activities of setup cost reduction.

For w_0, this threshold can be interpreted as the given product weight of the previous generation or a physical upper bound of the product weight that still maintains commercial viability to customers.

Assumption 3.3   *Manufacturing cost $c_0(w) = a_0 w$ ($/unit) where $a_0 > 0$.*

Electronic industries (e.g., single use cameras, copiers, CRT's) are generally equipped with highly automated production lines [7][9], and materials cost is the key driver of the manufacturing cost [20][21]. Direct Technologies estimated the manufacturing cost of air conditioners by

simply doubling the materials cost [22]. Furthermore, the linear cost structure enables us to develop a first-cut analysis and facilitates the analytic results. This functional form can be easily extended to other structures such as functions involving a quadratic term with respect to the product weight.

Assumption 3.4   *The collector is reimbursed at a rate of b ($/lb) by the manufacturer for used products.*

This assumption is based on the existing legislations/regulations and industrial practices. In California, a recycler shall pay a collector the Standard Statewide Combined Recovery Payment Rate (0.16 $/lb since July 1, 2008) for all covered electronic products transferred to the recycler [23]. In Connecticut, weight-based price is set in E-Waste laws for collectors to get reimbursed by manufacturers [24]. As for industrial practices, Kodak reimburses collectors $0.75/lb for the used single-use cameras regardless of the composition and brand [25]. Atasu et al. [13] also claimed that, since the WEEE recycling target is based on the product weight, cost allocation between manufacturers is currently weight-based, and is managed at best by sampling the collected products.

Furthermore, the economies of scale cost structure is particularly appropriate in modeling drop-off collection strategy. The collector invests in making customers become more aware of the collection program so that more customers drop off their used products at collection sites [26]. We also make a simplifying assumption that other operational costs of collecting activities are negligible.

Assumption 3.5   *Collection rate investment function $c(\tau) = -K\ln(1-\tau)$ ($) where $K > 0$.*

The collector invests in making customers aware of the take-back program (e.g., advertising and/or educational campaign) [26], and the investment is quantity-independent. Recall that the collection rate τ is the fraction of the sold products that are collected. One can think of τ as the response of customers to the investing activities. To incorporate the diminishing returns (from a lower to a higher τ) to investment [11], we utilize an exponential function $\tau = 1 - e^{-C/K}$ where K is a positive scaling parameter and C is the amount of investment. Similar response functions are frequently observed in advertising literature [27] characterizing customers' responsiveness to the advertising efforts. Hence, the investment function of the collection rate can be derived as in Assumption 3.5.

Assumption 3.6   *Each returned used product is recycled for the original purpose, resulting in a benefit of $\delta(w) = \Delta w$ ($/unit) where $0 < \Delta < a_0$.*

In recent years, there has been a growing emphasis on product recycling as a profitable process to reduce materials consumption and conserve energy. Recycling process converts used products into raw materials and energy, and can be profitable in cases such as cell phones, CRT's, and products with metal volume [9]. For instance, toner cartridge recycling is a 3 billion dollar industry [1]. HMR Solution claimed a nearly 100% recycling rate for CRT's, and reused the recycled glass, steel, plastics, and rare materials (e.g., tin, gold and palladium) [14]. Panasonic also utilized the recycled glass in producing new CRT's [3].

Here recycling benefit is defined as the difference between the value of recycled materials and the recycling cost. Due to the nature of recycling process, it is reasonable to assume that the value of recycled materials is proportional to the product weight. As for recycling cost, Shih et al. [28] claimed that the recycling cost of electronic products (e.g., computers) mainly de-

pends on the disassembly time, and estimated that the disassembly time is proportional to the product weight. Jeong and Lee [29] also pointed out that the recycling cost of LCD panels is proportional to the product weight. Furthermore, some articles claimed that the materials cost is the largest cost driver in e-waste recycling [30], and the recycling cost is closely allied to the product weight [31].

We note that, Δ here represents the marginal recycling benefit, and will be extensively utilized in the sensitivity analysis later. Also, the latter part of this assumption $0 < \Delta < a_0$ implies that the recycling process is profitable and the recycling benefit is less than the manufacturing cost (due to imperfect recycling and recycling cost).

Assumption 3.7   *Linear demand function $D(p) = \beta - \gamma p$ where p ($/unit) is the price and $\beta, \gamma > 0$.*

Linear demand functions are widely utilized in the supply chain [11] and economics literature [32] to facilitate the mathematic tractability and as a first order approximation.

Assumption 3.8   *The uncollected used products are disposed at a landfill.*

All in all, the uncollected used products (i.e., collection leakage) will end up at landfills [33][34]. In this paper, we assume that the environmental impact of the supply chains is measured by the landfill quantity (i.e., the total weight of the products sent to the landfill) *L*, and $L = w(1-\tau)D(p)$.

Assumption 3.9   *In all the models of this paper, the optimal/equilibrium decisions are interior solutions. Specifically, $\beta > \sqrt{8\gamma s}$ and $\Delta > \dfrac{Ka_0}{s}$ (or $\Delta + \sigma > \dfrac{K(a_0 + \alpha)}{s}$ for model with government subsidy and fee).*

See Appendix A for proof. by excluding pathological cases of boundary solutions, we intend to focus on the most relevant and interesting cases of our models. This assumption implies that the interiority of the optimal/equilibrium solutions can be guaranteed by a sufficiently large potential market size and recycling benefit. Also, this assumption implies that $S > K$ (according to Assumption 3.6).

Given these assumptions, we now proceed to the supply chain models as follows.

3.2.1   Non-Recycling Scenario (NR)

Without recycling, the collector is not considered in the NR model. Hence, the manufacturer decides the selling price p and the product weight w to maximize the profit Π_{NR}^{M}. Throughout this paper, Π_{i}^{j} will denote the profit of player j in supply chain scenario i.

$$\max_{p,w} \Pi_{NR}^{M} = (\beta - \gamma p)(p - a_0 w) - S\ln(w_0 / w) \tag{3.1}$$

The optimal solutions are $p_{NR} = \dfrac{3\beta - \sqrt{\beta^2 - 8\gamma S}}{4\gamma}$ and $w_{NR} = \dfrac{\beta - \sqrt{\beta^2 - 8\gamma S}}{2\gamma a_0}$. Since no product is taken back and recycled, the landfill quantity $L_{NR} = w_{NR}(\beta - \gamma p_{NR}) = S / a_0$ and the manufacturer's profit is

$$\Pi_{NR} = \frac{(\beta + \sqrt{\beta^2 - 8\gamma S})^2}{16\gamma} + S\ln\frac{\beta - \sqrt{\beta^2 - 8\gamma S}}{2\gamma a_0 w_0}$$

3.2.2   Centralized Supply Chain with Recycling (CR)

In the CR model, we assume that the manufacturer and the collector are vertically integrated, and all the decisions are made by a central planner with the objective of maximizing total supply chain profit. Centralized models have been extensively utilized as a benchmark against decentralized models for the comparative studies of economic factors such as price and profit, as well as environmental factors such as collection rate [13]. The relevant decision variables for

the central planner are the selling price p, the product weight w, and the collection rate τ. The profit maximization problem is

$$\max_{p,w,\tau} \Pi_{CR} = (\beta - \gamma p) \cdot [p - (a_0 - \tau\Delta)w] + K\ln(1-\tau) - S\ln(w_0/w) \tag{3.2}$$

The optimal solutions are $p_{CR} = \dfrac{3\beta - \sqrt{\beta^2 - 8\gamma S}}{4\gamma}$, $w_{CR} = \dfrac{(S-K)(\beta - \sqrt{\beta^2 - 8\gamma S})}{2\gamma S(a_0 - \Delta)}$, and $\tau_{CR} = 1 - \dfrac{K(a_0 - \Delta)}{\Delta(S-K)}$. Accordingly, the landfill quantity $L_{CR} = w_{CR}(1 - \tau_{CR})(\beta - \gamma p_{CR}) = K/\Delta$, and the total supply chain profit is

$$\Pi_{CR} = \frac{(\beta + \sqrt{\beta^2 - 8\gamma S})^2}{16\gamma} + K\ln\frac{K(a_0 - \Delta)}{\Delta(S-K)} + S\ln\frac{(S-K)(\beta - \sqrt{\beta^2 - 8\gamma S})}{2\gamma S w_0(a_0 - \Delta)}$$

3.2.3 Decentralized Supply Chain with Recycling (DR)

In the DR model, we assume that the manufacturer is the leader and the collector is the follower in a Stackelberg game. The assumption of a dominant manufacturer is frequently observed in supply chain literature and is based on the belief that downstream supply chain members such as collectors are often smaller in size and operate in specific local markets [40]. Also, we are assuming that the core competence of the manufacturer is not in collecting the used products [11][26]. Hence, given the selling price and the product weight, the collector optimally determines the collection rate via

$$\max_{\tau} \Pi_{DR}^C = bw\tau(\beta - \gamma p) + K\ln(1-\tau) \tag{3.3}$$

The best response function is $\tau^*(p,w,b) = 1 - \dfrac{K}{bw(\beta - \gamma p)}$. Considering the collector's best response, the manufacturer optimally determines the selling price, the product weight, and the buyback price via

$$\max_{p,w,b} \Pi_{DR}^M = (\beta - \gamma p) \cdot [p - (a_0 - \Delta + b)w] - K\Delta/b + K - S\ln(w_0/w) \tag{3.4}$$

The manufacturer's equilibrium solutions are $p_{DR} = \dfrac{3\beta - \sqrt{\beta^2 - 8\gamma S}}{4\gamma}$, $w_{DR} = \dfrac{2S(\beta - \sqrt{\beta^2 - 8\gamma S})}{\gamma[\sqrt{K\Delta} + \sqrt{K\Delta + 4S(a_0 - \Delta)}]^2}$,

and $b_{DR} = \dfrac{K\Delta + \sqrt{K^2\Delta^2 + 4SK\Delta(a_0 - \Delta)}}{2S}$. Substituting the manufacturer's solutions into the collector's

best response function, we have the collection rate $\tau_{DR} = 1 - \dfrac{K}{2S} - \dfrac{\sqrt{K^2\Delta^2 + 4SK\Delta(a_0 - \Delta)}}{2S\Delta}$. The landfill

quantity $L_{DR} = w_{DR}(1 - \tau_{DR})(\beta - \gamma p_{DR}) = \dfrac{2KS}{K\Delta + \sqrt{K^2\Delta^2 + 4SK\Delta(a_0 - \Delta)}}$, and the manufacturer's and collector's

profits are

$$\Pi_{DR}^M = \frac{(\beta + \sqrt{\beta^2 - 8\gamma S})^2}{16\gamma} + K - \frac{2SK\Delta}{K\Delta + \sqrt{K^2\Delta^2 + 4SK\Delta(a_0 - \Delta)}} + S\ln\frac{2S(\beta - \sqrt{\beta^2 - 8\gamma S})}{\gamma w_0[\sqrt{K\Delta} + \sqrt{K\Delta + 4S(a_0 - \Delta)}]^2} \tag{3.5}$$

$$\Pi_{DR}^C = \frac{(2S - K)\Delta - \sqrt{K^2\Delta^2 + 4SK\Delta(a_0 - \Delta)}}{\Delta[K\Delta + \sqrt{K^2\Delta^2 + 4SK\Delta(a_0 - \Delta)}]} + K\ln\frac{K\Delta + \sqrt{K^2\Delta^2 + 4SK\Delta(a_0 - \Delta)}}{2S\Delta} \tag{3.6}$$

3.2.4 Decentralized Supply Chain with Recycling and Government Subsidy/Fee (DR-G)

The supply chain scenario of the DR-G model is exactly the same as the DR model except that

we are considering the government participation in the form of subsidy and fee (see Figure 2).

Assumption 3.10 *In the DR-G model, under the framework of revenue-neutrality, the gov-*

ernment provides a subsidy of σ ($/lb) to the collector for the collecting of used products, and

receives a fee of α ($/lb) from the manufacturer for the products sold.
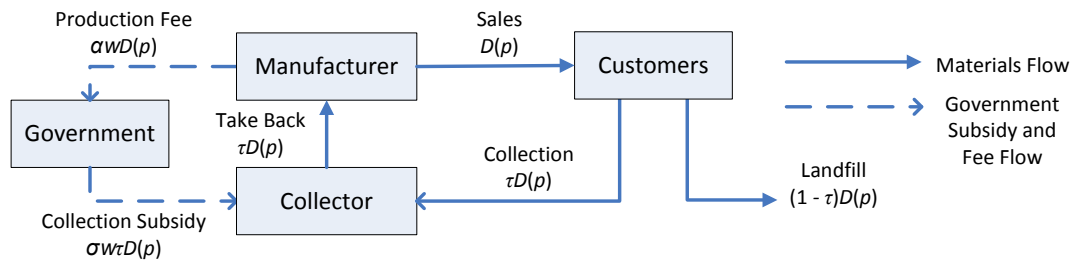
Figure 2. Decentralized Supply Chain with Government Subsidy and Fee

In California, collectors are subsidized \$0.2/lb for the collection of used video display devices [35]. In Nebraska, manufacturers are responsible to pay the State Agency of Natural Resources \$0.5/lb for the covered electronic devices based on annual statewide sales [36]. By revenue-neutrality, we mean that the financial gains from fees balance the financial expenses of subsidies so that government is financially neutral (i.e., $\alpha w D(p) = \sigma \tau w D(p)$). A revenue-neutral carbon tax has been implemented in British Columbia since 2008, and the tax revenue will be returned through tax reductions [37]. In the literature, Palmer and Walls [38] utilized a tax/subsidy combination in which producers pay a per-pound tax and collectors of used products receive a per-pound subsidy. Furthermore, Mrozek [39] argued that the deposit/refund systems must be implemented in a way that the revenue is neutral to achieve the maximum efficiency. The entire operation of DR-G model is depicted in Figure 2.

Given the selling price and collection rate, the collector optimally determines the collection rate via

$$\max_{\tau} \Pi^C_{DR-G} = \tau w(b+\sigma)(\beta - \gamma p) + K\ln(1-\tau) \tag{3.7}$$

The collector's best response function is $\tau^*(p,w,b) = 1 - \dfrac{K}{w(b+\sigma)(\beta - \gamma p)}$. Considering the best response function, the manufacturer maximizes his profit via

$$\max_{p,w,b} \Pi^M_{DR-G} = (\beta - \gamma p) \cdot \left[p - (a_0 - \Delta + b + \alpha)w \right] - K(\Delta - b)/(b+\sigma) - S\ln(w_0/w) \tag{3.8}$$

The solutions are $p_{DR-G} = \dfrac{3\beta - \sqrt{\beta^2 - 8\gamma S}}{4\gamma}$, $w_{DR-G} = \dfrac{2S(\beta - \sqrt{\beta^2 - 8\gamma S})}{\gamma[\sqrt{K(\Delta+\sigma)} + \sqrt{K(\Delta+\sigma) + 4S(a_0 - \Delta + \alpha - \sigma)}]^2}$, and

$b_{DR-G} = \dfrac{K(\Delta+\sigma) - 2S\sigma + \sqrt{K^2(\Delta+\sigma)^2 + 4SK(\Delta+\sigma)(a_0 - \Delta + \alpha - \sigma)}}{2S}$. Substituting these solutions into the best

response function, we have $\tau_{DR\text{-}G} = 1 - \dfrac{K}{2S} - \dfrac{\sqrt{K^2(\Delta+\sigma)^2 + 4SK(\Delta+\sigma)(a_0 - \Delta + \alpha - \sigma)}}{2S(\Delta+\sigma)}$. Accordingly, the

landfill quantity $L_{DR\text{-}G} = w_{DR\text{-}G}(1 - \tau_{DR\text{-}G})(\beta - \gamma p_{DR\text{-}G}) = \dfrac{2KS}{K(\Delta+\sigma) + \sqrt{K^2(\Delta+\sigma)^2 + 4SK(\Delta+\sigma)(a_0 - \Delta + \alpha - \sigma)}}$. The man-

ufacturer's and the collector's profits are

$$\Pi^M_{DR\text{-}G} = \frac{(\beta + \sqrt{\beta^2 - 8\gamma S})^2}{16\gamma} + K - \frac{2SK(\Delta+\sigma)}{K(\Delta+\sigma) + \sqrt{K^2(\Delta+\sigma)^2 + 4SK(\Delta+\sigma)(a_0 - \Delta + \alpha - \sigma)}}$$

$$+ S \ln \frac{2S(\beta - \sqrt{\beta^2 - 8\gamma S})}{\gamma w_0 [\sqrt{K(\Delta+\sigma)} + \sqrt{K(\Delta+\sigma) + 4S(a_0 - \Delta + \alpha - \sigma)}]^2} \tag{3.9}$$

$$\Pi^C_{DR\text{-}G} = \frac{(2S - K)(\Delta+\sigma) - \sqrt{K^2(\Delta+\sigma)^2 + 4SK(\Delta+\sigma)(a_0 - \Delta + \alpha - \sigma)}}{(\Delta+\sigma)[K(\Delta+\sigma) + \sqrt{K^2(\Delta+\sigma)^2 + 4SK\Delta(a_0 - \Delta + \alpha - \sigma)}]}$$

$$+ K \ln \frac{K(\Delta+\sigma) + \sqrt{K^2(\Delta+\sigma)^2 + 4SK(\Delta+\sigma)(a_0 - \Delta + \alpha - \sigma)}}{2S(\Delta+\sigma)} \tag{3.10}$$

Based on the optimal/equilibrium solutions summarized in Table 1, we performed the sensitivity analysis based on marginal recycling benefit and collecting subsidy, as well as comparison analysis among three CLSC scenarios. Recycling benefit is the critical driver of recycling profitability [12], and accordingly, is the economic motivation behind the manufacturer's recycling investment and collector's take back investment [11][13].

Proposition 3.1 $\Pi_{CR} > \Pi_{NR}$ and $\Pi^M_{DR} > \Pi_{NR}$.

See Appendix B for proof. If $\Delta < Ka_0/S$, the value of recycled materials will not cover the take back cost and the recycling cost. Also, if $\Delta > Ka_0/S$, the manufacturer's profits in the DR model and the total supply chain profit in the CR model are greater than the manufacturer's profit in the NR model.

Proposition 3.2 In the DR model, $\dfrac{\partial w_{DR}}{\partial \Delta} > 0$, $\dfrac{\partial \tau_{DR}}{\partial \Delta} > 0$.

See Appendix C for proof. This proposition states that the increase (decrease) of the marginal

recycling benefit Δ will result in both higher (lower) levels of product weight and collection rate

(see Figure 3 based on numerical examples).
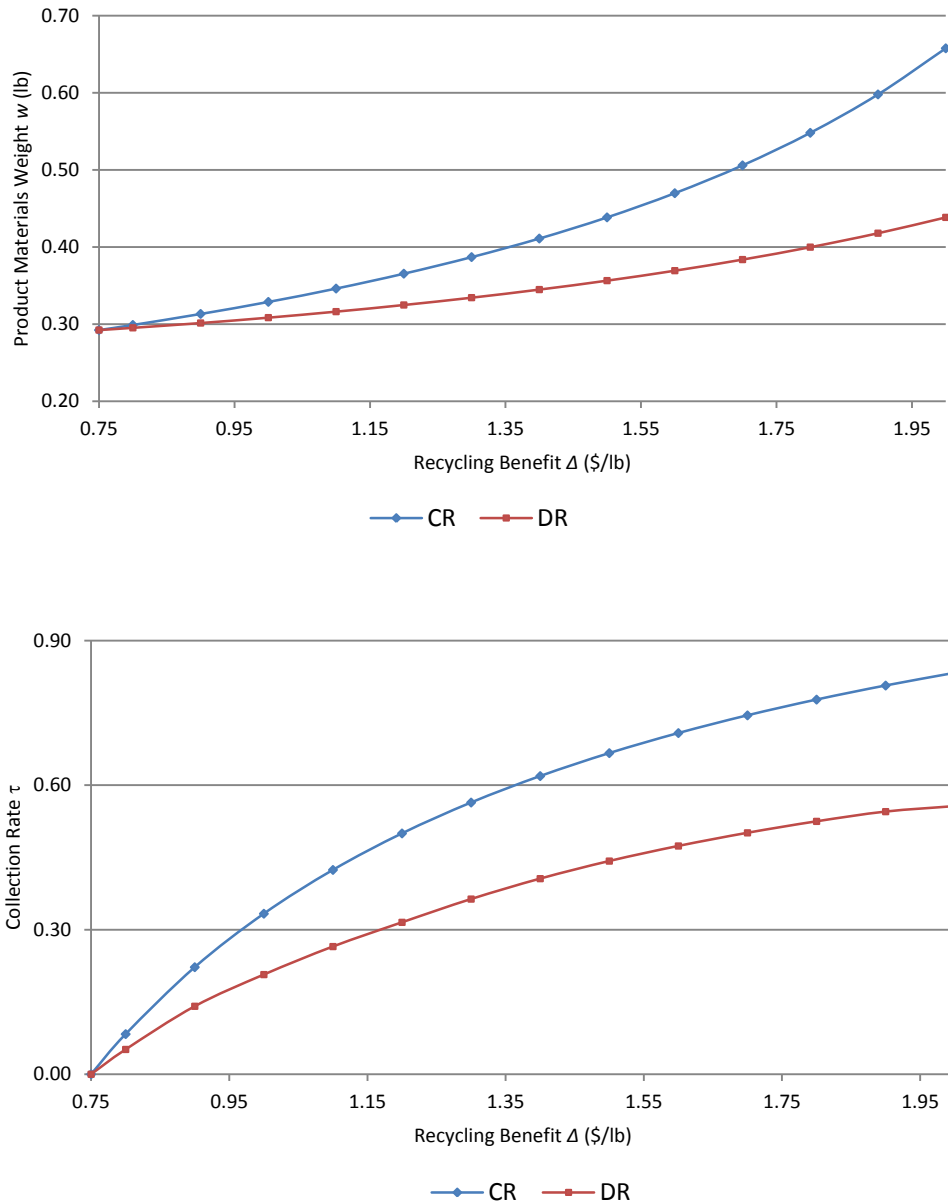
Figure 3. Variation of Production Weight and Collection Rate with Marginal Recycling Benefit

Intuitively, as Δ increases, there is an incentive for the manufacturer to pursue a higher level

of the collection rate so as to gain more benefits from recycling process. Hence, the manufac-

turer incentivizes the collector to take back more products via increasing the buyback price and reduce the investment in the product weight reduction. Hence, decision makers may not be able to pursue both higher level of the collection rate and lower level of the product weight simultaneously, and it is advised to examine the negative impact on the other factor before any efforts on improving one environmental factor.

Proposition 3.3   *In the DR model, $\frac{\partial L_{DR}}{\partial \Delta} < 0$ if $\frac{Ka_0}{S} < \Delta < \frac{a_0\sqrt{S}}{2\sqrt{S}-\sqrt{K}}$, and $\frac{\partial L_{DR}}{\partial \Delta} > 0$ if $\Delta > \frac{a_0\sqrt{S}}{2\sqrt{S}-\sqrt{K}}$.*

See Appendix D for proof. This proposition indicates that, if the marginal recycling benefit is relatively low, the landfill quantity decreases with the marginal recycling benefit, and if the marginal recycling benefit is relatively high, the landfill quantity increases with the marginal recycling benefit (see Figure 4 based on numerical examples).
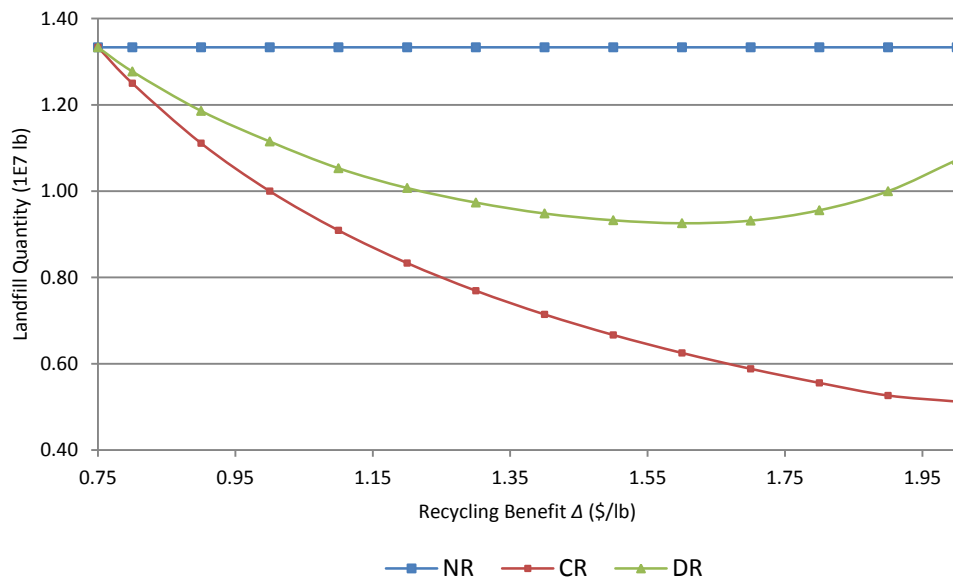
Figure 4. Variation of the Landfill Quantity with Marginal Recycling Benefit

Intuitively, when the marginal recycling benefit is relatively high, both the product weight and the collection rate are also relatively high (see Proposition 3.2). Accordingly, the effect of diminishing returns to the collection investment dominates that of the product weight reduc-

tion investment. As a result, the marginal increase of the product weight with respect to Δ dominates the marginal increase of the collection rate with respect to Δ (which can be observed in Figure 3), which leads to a higher level of the landfill quantity. In this sense, the increase of the marginal recycling benefit may not always favor the reduction of landfill quantity, and hence, should not be encouraged unconditionally. This probably unintended consequence implies that the decision makers must view the critical environmental factors in totality before any effort to improve a particular set of environmental factors.

Proposition 3.4 *In the NR, CR, and DR models, given the same values of parameters, the product weight, the collection rate, and the landfill quantity are related as follows:*

$$w_{NR} < w_{DR} < w_{CR}, \ \tau_{DR} < \tau_{CR}, \ L_{NR} > L_{DR} > L_{CR}$$

See Appendix E for proof. This proposition demonstrates that the centralized model dominates the decentralized model in terms of higher collection rate and lower landfill quantity due to its ability to coordinate and avoid double marginalization [11]. This is because the central planner can improve the collection rate by coordinating the forward and reverse channel decisions via a single two-part tariff [11]. However, due to the inherent conflict between the investments in product weight reduction and collection rate enhancement, centralization also results in a higher product weight relative to the decentralization case (see Figure 3). Moreover, product recycling is always beneficial to lessening the environmental burdens in terms of less landfill quantity.

Thus far we have examined the key insights and implications of the supply chain models without government intervention. Let us proceed to examine such insights and implications of the supply chain models with government intervention.

Proposition 3.5   *In the DR-G model, $\frac{\partial w_{DR\text{-}G}}{\partial \sigma} > 0$, $\frac{\partial \tau_{DR\text{-}G}}{\partial \sigma} > 0$.*

  See Appendix F for proof. Similar to Proposition 3.2, this proposition states that the increase (decrease) of the collecting subsidy σ will result in both higher (lower) levels of product weight and collection rate. Intuitively, as σ increases, there is an incentive for the collector to pursue a higher level of the collection rate so as to gain more benefits from recycling process. From the manufacturer's perspective, with a higher level of the returned quantity, there is an incentive to reduce the investment in product weight reduction so as to take more advantage of the recycling benefit.

    Given this knowledge, policy makers should be aware of both positive and negative consequences on the environmental factors before implementing the subsidy and fee. Furthermore, it is extremely desirable to investigate the corresponding environmental consequence involving these two factors.

Proposition 3.6   *In the DR-G model, $\frac{\partial L_{DR\text{-}G}}{\partial \sigma} < 0$ if $\sigma < \frac{\sqrt{S}(a_0 + \alpha)}{2\sqrt{S} - \sqrt{K}} - \Delta$; $\frac{\partial L_{DR\text{-}G}}{\partial \sigma} > 0$ if $\sigma > \frac{\sqrt{S}(a_0 + \alpha)}{2\sqrt{S} - \sqrt{K}} - \Delta$*

    See Appendix G for proof. Similar to Proposition 3.3, this proposition indicates that, if the collecting subsidy is relatively low, the landfill quantity decreases with the collecting subsidy, and if the collecting subsidy is relatively high, the landfill quantity increases with the collecting subsidy (see Figure 5 based on numerical examples). In this sense, the increase of the collecting subsidy may not always favor the reduction of landfill quantity, and hence, should not be encouraged unconditionally. We refer the readers to the illustration of Proposition 3.3 for the essential ideas of the intuition of this proposition.

This proposition implies that, to avoid unintended consequences, policy makers must view the critical environmental factors in totality before any financial instrument encouraging a particular set of the environmental factor(s).

Due to the similarity between the DR and DR-G model, it is desirable to compare them in terms of the product weight, collection rate, and landfill quantity. Meanwhile, the CR model is also utilized as a benchmark to investigate whether and how the environmental performance in the centralized model can be achieved in the decentralized model.

Proposition 3.7   *In the DR and DR-G model, given the same values of parameters, the product weight and the collection rate are related as follows:* $w_{DR} < w_{DR\text{-}G}$, $\tau_{DR} < \tau_{DR\text{-}G}$.

See Appendix H for proof. This proposition demonstrates that, government participation in the form of collection subsidy and production fee will result in both higher levels of the product weight and the collection rate. This implies that incentives for product collection also discourage manufacturer's efforts on product weight reduction, which may not be the original intention of policy makers. In this sense, it is interesting to further compare the total environmental consequence in terms of the landfill quantity.

Proposition 3.8   *In the DR-G model,*

$$L_{DR\text{-}G} < L_{CR} \ \ if \ \ \frac{2S\Delta(S\Delta - Ka_0)}{K(2Sa_0 - K\Delta)} < \sigma < \frac{\Delta\sqrt{K^2\Delta^2 + 4SK\Delta(a_0 - \Delta)} - K\Delta^2}{2K(a_0 - \Delta)} - \Delta \ ;$$

$$L_{CR} < L_{DR\text{-}G} < L_{DR} \ \ if \ \ 0 < \sigma < \frac{2S\Delta(S\Delta - Ka_0)}{K(2Sa_0 - K\Delta)}$$

$$or \ \ \frac{\Delta\sqrt{K^2\Delta^2 + 4SK\Delta(a_0 - \Delta)} - K\Delta^2}{2K(a_0 - \Delta)} - \Delta < \sigma < \frac{S(a_0 - \Delta) + \sqrt{K^2\Delta^2 + 4SK\Delta(a_0 - \Delta)}}{K}$$

$$L_{DR\text{-}G} > L_{DR} \ \ if \ \ \sigma > \frac{S(a_0 - \Delta) + \sqrt{K^2\Delta^2 + 4SK\Delta(a_0 - \Delta)}}{K}$$

See Appendix I for proof. Proposition 3.8 demonstrates the significance of the subsidy on the total environmental impact under the revenue-neutrality framework. Specifically, by choosing an appropriate value of subsidy, the landfill quantity in the centralized model can be achieved or even further reduced in the decentralized model (see Figure 5). Nevertheless, on the other hand, a sufficiently high value of the subsidy may also result in a higher level of the landfill quantity relative to the non-subsidy case (see Figure 5). This implies that implementing a subsidy/fee instrument into the CLSC's may not always be environmentally viable. Given these findings, the pricing of subsidy/fee becomes important, and it is highly advised to examine the whole supply chain information before any financial instrument is offered so as to realize high environmental efficiency (less landfill quantity) and avoid unintended negative environmental consequences (more landfill quantity).
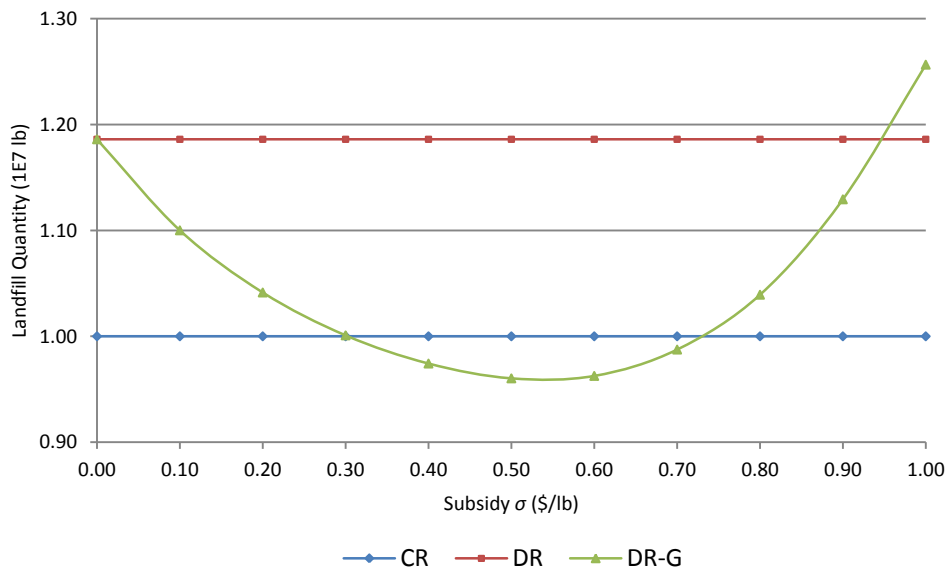
Figure 5. Variation of the Landfill Quantity with the Collecting Subsidy

Furthermore, we note that the selling prices in the decentralized models are identical to the selling price in the centralized model. This is because there is no double-marginalization in the forward supply chain when the manufacturer directly sells the products to customers.

3.3   Managerial Insights and Policy Implications

The analytic results in this paper demonstrate that: (i) There is an inherent conflict between product weight reduction and collection rate improvement. That is, conditions (e.g., increase in the marginal recycling benefit) leading to a higher level of the collection rate may also result in a higher level of the product weight. (ii) As a result, efforts on improving one environmental factor may negatively impact the other factor, which, in some cases, will result in an unintended higher level of the landfill quantity. In other words, under certain conditions, the environment is worse off as product recycling becomes more attractive. In this sense, decision makers are advised to view the impacts on both environmental factors as a whole and examine the total environmental consequence before any efforts. (iii) Meanwhile, a policy maker is advised to be cautious when implementing a subsidy/fee mechanism to CLSC's under the framework of revenue-neutrality (i.e., subsidies balance fees). For example, if the subsidy is sufficiently high, the CLSC with subsidy/fee may be inferior to the non-subsidy/fee case with respect to the landfill quantity. Hence, the pricing of subsidy/fee becomes critically important to achieve high environmental efficiency and avoid any unintended negative environmental consequences.

CHAPTER 4

REMANUFACTURING DECISION WITH REMANUFACTURING COST UNCERTAINTY

4.1   Background and Objectives

Among the existing sustainable processes for end-of-life products, remanufacturing provides an opportunity for firms from various industries to mitigate the environmental impacts while maintaining economic viability. From decision/policy makers' perspective, there are substantial opportunities to reap the economic and environmental benefits of the remanufacturing, such as reverse supply chain designs and acquisition strategies. Among these opportunities, remanufacturing timing is especially important for firms providing services or leasing products [30] in the sense that the product life is determined by internal process management and/or leasing contracts. We note that, for this product life, the physical condition (e.g., component abrasion) deteriorates over time on average, which complicates the decision making process on remanufacturing timing. This is supported by the evidence that the quality of returned products to manufacturers in general varies vastly [31].

With this knowledge, it is highly desirable to understand the economically rational decisions on the remanufacturing timing under remanufacturing cost uncertainties. Furthermore, for policy makers, especially in the areas of governmental subsidies and fees, it is critical to understand how appropriate policies could influence the remanufacturing decisions so as to reduce the environmental impacts.

In this chapter, we consider a firm that leases a single product to a service provider, and the firm is also responsible for the maintenance of the product. For instance, Xerox leases photocopiers to Staples which provides copying services to customers. Throughout a product's lifecy-

cle, as the maintenance cost increases (due to component-deterioration), at any time point, the firm has an option to terminate the life of the product and remanufacture it. During the remanufacturing process, depending on the physical condition, components are either reused or replaced and disposed. After the remanufacturing process, the product is restored to a like-new condition and re-leased to the service provider.

We note that the flexibility on remanufacturing described above lends itself to a real options approach. That is, the firm has the right, but not the obligation, to make changes to its business project under uncertainty [41]. Moreover, in the extended scenario, government participation is incorporated into our model in the form of remanufacturing subsidy and disposal fee. Under these modeling frameworks, this study attempts to

(1) Formulate and analyze mathematical models of the remanufacturing decision from a real options perspective when the remanufacturing cost follows a Geometric Brownian motion (GBM) process.

(2) Provide managerial insights by examining how the remanufacturing cost uncertainty and other relevant factors influence the exercise of the remanufacturing option.

(3) Derive policy implications by examining the impacts of the remanufacturing subsidy and disposal fee on the firm's remanufacturing decision as well as on the environment.

(4) Provide guidelines for the government regarding how policies should be adjusted in response to the variation of remanufacturing cost uncertainty so as to mitigate the environmental impacts.

To our knowledge, what distinguishes this article from the extant literature is that (i) this is the first quantitative study investigating the economic rationale of remanufacturing decision

from a real options perspective; (ii) the product life and the reuse/disposal fraction are considered simultaneously, with the objective of examining the environmental impacts resulting from the remanufacturing decision; (iii) the viability of governmental policies is measured by environmental performances instead of total surplus.

The critical findings of this paper include: (i) The uncertainty of remanufacturing cost is the driver for decision makers to prolong the product life and defer the exercise of remanufacturing option because the uncertainty increases the value of holding the flexibility; (ii) Increasing the remanufacturing subsidy (disposal fee) incentivizes decision makers to exercise the remanufacturing option earlier (later) because the subsidy reduces (increases) the cost of exercising the option now. That is, the value of waiting is diminished; (iii) An increase of the remanufacturing subsidy (disposal fee) may result in more industrial wastes because it entails a shorter product life (lower reuse fraction of materials); (iv) As the remanufacturing cost becomes more uncertain, the policy maker is advised to increase the remanufacturing subsidy or decrease the disposal fee because these adjustments balance the higher level of landfill disposal stemmed from the higher uncertainty. We hope that these findings, as the first research findings on the remanufacturing decisions via a real options approach will stimulate relevant discussions among industrial practitioners, governmental regulators, environmental groups, as well as academics.

4.2   Models and Analyses

4.2.1   Basic Model

In this section, let us consider a firm that leases a product to a service provider, and then the service provider utilizes the services derived from the product to serve customers. Meanwhile, the firm provides the maintenance service for the product to preserve the product in a reliable

operating condition. Such leasing and maintenance contracts exist in a variety of industries, such as photocopier leasing contract between Xerox and Staples as well as mail processing machine leasing contracts between Pitney Bowes and USPS. Furthermore, the firm has an option to terminate the life of the product and remanufacture it. The product is restored to like-new condition during the remanufacturing process, and re-leased to the service provider. Prior to modeling the remanufacturing decision process, we will first provide the key assumptions of our models.

Assumption 4.1   *The remanufacturing period is assumed to be negligible.*

This simplifying assumption is made to focus on the remanufacturing decision without diluting our attention to various transient issues such as how to model the behavior of the manufacturer as well as the service provider during the remanufacturing [42].

Assumption 4.2   *There exists a leasing contract between the firm and the service provider with a leasing price of P ($/unit product) at any time point. By this contract, the firm is responsible for the maintenance of the product to preserve it in a reliable operating condition. The contract term is sufficiently long that the expiration of the contract is not considered in the models.*

Assumption 4.3   *The remanufacturing cost C ($/unit product), at any time point, follows a GBM process, i.e., $dC = \mu C dt + \sigma C dz$ where $\mu, \sigma > 0$. Here μ and σ are the instantaneous growth rate (%/unit time) and volatility, respectively. dz is the increment of a standard Wiener process ($dz = \varepsilon_t \sqrt{dt}$ where $\varepsilon_t \sim N(0,1)$). Furthermore, at the beginning of the product life, the remanufacturing cost is positive, denoted by c_0.*

Technically, we note that the source of cost uncertainty in this paper is the volatility of GBM process. Remanufacturing is a process of restoring the end-of-life products to like-new condi-

tion by replacing the parts that are affecting the performance of the whole product [44]. At the beginning of the process, for each component, the reuse potential is assessed based on the component's overall life and the used life under the operating conditions [45]. Typically, in a product's lifecycle, the components subject to quality deterioration over time (i.e., functioning less efficiently than that at the beginning of the product life) [46]. For example, in photocopier, the abrasion of fast moving parts such as bushings is sensitive to the operating life [47]. Therefore, on average, the longer time a product is being operated before remanufacturing, the more components subject to replacement, which implies a higher remanufacturing cost.

Meanwhile, service demand (units of service/unit time) following a stochastic process in general and a GBM process in particular can be observed in the literature [48][49]. By a unit service, we mean a quantifiable measure of service such as a single paper copy in Staples or a load of clothes in a laundry. Hence, being sensitive to the serving frequency, the deterioration of components and product as a whole are stochastic with respect to the operating time [50].

As a classical stochastic process with deterministic growth rate, GBM process is utilized in our models to characterize the variation of remanufacturing cost over operating time while facilitating the analytical investigation of the models. We note that modeling cost factor as a GBM process is not new [51][52]. Furthermore, Zambujal-Oliveira and Duque [53] modeled the salvage value of end-of-life products as a GBM process where the salvage value can be roughly interpreted as the remanufacturing cost saving (i.e., the cost difference between manufacturing a new product and remanufacturing a used product) [11]. Therefore, the remanufacturing cost uncertainty could also result from the fluctuation of market price for raw materials or components that are replaced in the remanufacturing process, which is widely modeled as a GBM pro-

cess in the literature. We also note that the remanufacturing cost may decrease because the prices for the components that are replaced during the remanufacturing process may drops.

Furthermore, we assume a positive remanufacturing cost at the beginning of the product life to avoid the trivial solution of exercising remanufacturing option without operating the product. This positive value can interpreted as the costs associated with cleaning, testing, and disassembly procedures [54].

Assumption 4.4   *By the leasing contract, the firm is responsible for maintaining the product in a reliable operating condition. At any time point, the maintenance cost M ($/unit product) is proportional to the instantaneous remanufacturing cost, i.e.,* $M(C) = \alpha C$ *where* $0 < \alpha < 1$.

The constant term in *M*(*C*) is not presented because, mathematically, any constant term can be incorporated into the leasing price without affecting the analytical results. Finally, $0 < \alpha < 1$ avoids the trivial solution of exercising the option at the beginning of the product life.

Assumption 4.5   *The number of products leased to service providers is sufficiently large so that the capital cost involved in remanufacturing process such as equipment cost is negligible for each product.*

Without considering the economies of scale for product transportation and installation, the decision process for the remanufacturing of each product is independent. Therefore, even though our models are based on a single product, they can be easily extended to multi-product models without affecting any analytical result. In this sense, as long as the number of products is large enough, the lump sum capital cost for each product is negligible compared to the remanufacturing cost, and hence, has negligible impact on the exercising of remanufacturing option. Furthermore, we assume that the cost of acquiring remanufacturing option is negligible. In

certain systems with small amount of inputs such as excavator remanufacturing at Case New Holland where equipment cost cannot be ignored, decision makers can compare this cost to the value of the remanufacturing option to decide whether or not it is optimal to acquire this flexibility.

Given these assumptions, the remanufacturing decision can be interpreted as an optimal stopping problem. For a single product, at any time point, the decision maker measures the remanufacturing cost and the net present value of the leasing project (will be referred as *project value* hereafter) to decide whether or not to exercise the remanufacturing option. The project value is defined as the sum of the discounted expected cash flow and the value of the remanufacturing option (or the value of flexibility).

Since the leasing price is fixed and the maintenance cost increases with time (Assumption 3.4), the project value decreases as product ages. Also, the remanufacturing cost increases with time. However, the project value tends to decrease faster than the remanufacturing cost because the increase of maintenance cost has a cumulative impact on the project value depreciation over time. Furthermore, through remanufacturing, a product is restored to like-new condition, and accordingly, the maintenance cost is restored to the lowest level (i.e., $M(C) = \alpha C_0$). Therefore, at certain time point, the remanufacturing cost balances the difference between the original project value (at the beginning of the product life with the lowest maintenance cost) and the instantaneous project value. Therefore, the remanufacturing cost at that time point becomes the threshold above which it is optimal to exercise the option. Furthermore, since a product is re-leased to the service provider after remanufacturing, there are infinite remanufacturing options with a common remanufacturing cost threshold to exercise. Without loss of gen-

erality, in this study, we target to resolve the problem with single remanufacturing option. As long as the remanufacturing option is not exercised, the project value *V* must satisfy the following Bellman's equation [59].

$$\rho V dt = (P - \alpha C)dt + E[dV | C] \tag{4.1}$$

where ρ is the discount rate per unit time, and $E[]$ is the operator of expectation value. The left-hand side of (4.1) is the return per unit time for holding the remanufacturing option. On the right-hand side, the first term is the immediate profit from holding the remanufacturing option, while the second term is the expected depreciation of the project value.

By Ito's Lemma [59], it can be verified that

$$dV = \frac{dV}{dC}(\mu C dt + \sigma C dz) + \frac{1}{2}\frac{d^2V}{dC^2}\sigma^2 C^2 dt \tag{4.2}$$

Substituting equation (4.2) into equation (4.1), we have

$$\frac{1}{2}\sigma^2 C^2 \frac{d^2V}{dC^2} + \mu C \frac{dV}{dC} - \rho V + P - \alpha C = 0 \tag{4.3}$$

To guarantee the convergence of the differential equation (4.3), we impose that $\mu < \rho$. Also, it can be verified that the project value function is given by (see Appendix J for proof)

$$V(C) = A_1 C^{\beta_1} - \frac{\alpha C}{\rho - \mu} + \frac{P}{\rho} \tag{4.4}$$

where $\beta_1 = \left[\frac{1}{2}\sigma^2 - \mu + \sqrt{\left(\frac{1}{2}\sigma^2 - \mu\right)^2 + 2\rho\sigma^2}\right] \Big/ \sigma^2 > 1$.

In equation (4.4), the first term is the value of the remanufacturing option, and the second (third) term is the discounted expected cost (revenue) flow. It can be observed that option value increases with the remanufacturing cost, and the discounted cash flow decreases with the remanufacturing cost.

Suppose C_R represents the remanufacturing cost threshold, then the project value function satisfies the following boundary conditions [59].

$$V(C_R) = V(C_0) - C_R \quad \Rightarrow \quad A_1 C_0^{\beta_1} - \frac{\alpha C_0}{\rho - \mu} - A_1 C_R^{\beta_1} + \frac{\alpha C_R}{\rho - \mu} - C_R = 0 \tag{4.5}$$

$$V'(C_R) = -1 \quad \Rightarrow \quad A_1 \beta_1 C_R^{\beta_1 - 1} - \frac{\alpha}{\rho - \mu} = -1 \tag{4.6}$$

Equation (4.5) is the value-matching condition and equation (4.6) is the smooth-pasting condition. The value-matching condition requires that, at the remanufacturing cost threshold, the expected value of an existing unit equals the expected NPV of a remanufactured one plus the value of embedded remanufacturing option. The smooth-pasting condition assures that C_R is the optimal exercise point by defining the continuance and smoothness of $V(C)$ at C_R.

Proposition 4.1   *There exists a unique solution for the remanufacturing cost threshold C_R.*

See Appendix K for proof. Even though there is no explicit closed-form solution for C_R, the following propositions can be derived by implicit function theorem.

Proposition 4.2   *Given $0 < \mu < \rho$, $\frac{\partial C_R}{\partial \sigma} > 0$ and $\frac{\partial C_R}{\partial C_0} > 0$.*

See Appendix L for proof. The interpretation of this proposition is given as follows: $\partial C_R / \partial \sigma > 0$ indicates that an increase in the volatility leads to an increase in the remanufacturing cost threshold. This is because, as the volatility increases, the value of the remanufacturing option also increases, and hence, it is more beneficial to hold the option for a higher remanufacturing cost. $\partial C_R / \partial C_0 > 0$ indicates that, as the fixed cost (i.e., associated with cleaning, testing, and disassembly) increases, the remanufacturing cost threshold increases. This is because a higher fixed cost decreases the new project value after remanufacturing, which incentivizes the firm to

defer the exercise of the remanufacturing option and wait for a higher remanufacturing cost threshold..

Thus far, we have performed analysis for the remanufacturing cost surrounding the remanufacturing decision. We now proceed to derive the expected life of the product. Suppose $F(C) = \ln C$, by Ito's lemma,

$$dF = \frac{dF}{dC}(\mu C dt + \sigma C dz) + \frac{1}{2}\frac{d^2 F}{dC^2}\sigma^2 C^2 dt = \frac{(\mu C dt + \sigma C dz)}{C} - \frac{\sigma^2 C^2 dt}{2C^2} = (\mu - \sigma^2/2)dt + \sigma dz \tag{4.7}$$

For any finite time period T, the change in $F(C)$ is normally distributed with a mean of $(\mu - \sigma^2/2)T$ and a variance of $\sigma^2 T$. Hence, the expected passage time from C_0 to C_R is $T = (\ln C_R - \ln C_0)/(\mu - \sigma^2/2)$. It can be observed that the variation of expected product life with the remanufacturing cost volatility depends on the parameter values [43]. Specifically, if $\mu > \sigma^2/2$, the expected life of the product increases with the remanufacturing cost volatility (i.e., $\partial T/\partial \sigma > 0$); if $\mu < \sigma^2/2$, the expected product life decreases with the remanufacturing cost volatility (i.e., $\partial T/\partial \sigma < 0$).

4.2.2 Extended Model with Remanufacturing Subsidy and Disposal Fee

In this subsection, we extend the basic model by incorporating government economic instruments in the form of remanufacturing subsidy and disposal fee as detailed in the following two assumptions.

Assumption 4.6 *There exists a subsidy S ($/unit product) for the remanufacturing of end-of-life products.*

In recent years, the usage of subsidies as an economic instrument to facilitate the remanufacturing of end-of-life products has significantly increased at all of local, state, and national

levels. For example, in California, remanufacturers are subsidized $0.28/lb for remanufacturing electronic products. In New York, the remanufacturers receive tax credits that are commensurate with the number of employees and/or the durability of capital investment. However, critical questions such as whether the existing subsidies are environmentally viable and how to best utilize these incentives to minimize the environmental impacts are rarely answered in academic articles or government reports. Therefore, to examine the environmental viability of the remanufacturing subsidy, we assume that such a subsidy will be reimbursed to the firm once the remanufacturing option is exercised.

Assumption 4.7 *The weight of a unit product is W (lb). In the remanufacturing process, for a unit product, $d(C)$ fraction (in weight) of the materials is disposed at a disposal fee of L ($/lb), the rest $1-d(C)$ fraction (in weight) of materials is reused.*

Here $d(C)$ is defined as the disposal fraction, and $1-d(C)$ is defined as the reuse fraction. During the remanufacturing process, the non-reusable components are disposed at landfill with a disposal fee [44]. Nowadays, both public and private landfills exist in the waste management market. While private landfills are profit-driven, public landfills are operated by local governments with the primary goal of reducing the landfill disposal, which entails a higher disposal fee relative to the private landfills [60]. However, our analytical results demonstrate that, a higher disposal fee may not result in less amount of wastes end up at landfill. This ramification cannot be the original intention of policy makers. Therefore, similar economic instruments should be carefully examined before implementation.

By this assumption, given the expected product life *T*, we are able to define the expected weight of disposal per unit time (will be referred as disposal rate hereafter), denoted by Q, as
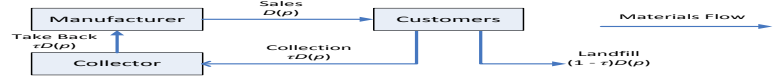
Equivalently, Q can be interpreted as the weight of material consumption per unit time. Throughout this paper, Q will be utilized as a critical measurement of the environmental impacts. By Assumptions 4.6 & 4.7, we have the new value-matching and smooth pasting conditions as follows.

$$V(C_R)=V(C_0)-C_R-d(C_R)WL+S \quad \Rightarrow \quad A_1 C_R^{\beta_1} - \frac{\alpha C_R}{\rho-\mu} - A_1 C_0^{\beta_1} + \frac{\alpha C_0}{\rho-\mu} + C_R + d(C_R)WL - S = 0 \qquad (4.9)$$

$$V'(C_R)=-1-d'(C_R)WL \quad \Rightarrow \quad A_1\beta_1 C_R^{\beta_1-1} - \frac{\alpha}{\rho-\mu} = -1-d'(C_R)WL \qquad (4.10)$$

Assumption 4.8   *The disposal fraction is linear to the remanufacturing cost, i.e.,* $d(C)=\gamma(C-C_0)+d_0$.

The disposal fraction is determined by the amount of worn-out components that are replaced in the remanufacturing process. $C - C_0$ represents the component-replacement cost in the remanufacturing process where C_0 represents the costs associated with cleaning, testing, and disassembly procedures. Moreover, term d_0 is associated with the parts that are mandated to be replaced in the remanufacturing process regardless of the physical conditions, such as packages and covers. For example, in the remanufacturing process of photocopies, plastic covers are mandated to be replaced with new ones in case that the crack on the old covers would affect the quality of the copy works [61].

We acknowledge that the weight of disposed materials may not be perfectly linear to the remanufacturing cost. However, this simplifying assumption facilitates the tractability of the analysis, and can be observed in the literature [62]. Given this assumption, in what follows, we

will provide a series of propositions characterizing the impacts of remanufacturing subsidy and disposal fee on the remanufacturing decision and the environment.

Proposition 4.3   *Given* $0 < \mu < \rho$, $\frac{\partial C_R}{\partial S} < 0$, $\frac{\partial C_R}{\partial L} > 0$, $\frac{\partial T}{\partial S} < 0$, *and* $\frac{\partial T}{\partial L} > 0$.

See Appendix M for proof. The interpretations of $\partial C_R / \partial S < 0$ and $\partial T / \partial S < 0$ are straightforward. As the remanufacturing subsidy increases, the cost of exercising the remanufacturing option decreases, which incentivizes the firm to exercise the option earlier. Accordingly, the remanufacturing cost threshold C_R becomes lower and the expected life T becomes shorter. As for $\partial C_R / \partial L > 0$ and $\partial T / \partial L > 0$, the increase of disposal fee implies a higher cost of exercising the option, resulting in a deference of remanufacturing.

In addition to the economic impacts on the firm's remanufacturing decision, from an environmental perspective, the remanufacturing subsidy and the disposal fee are influencing the disposal rate from two aspects: (i) an increase of the remanufacturing subsidy (disposal fee) will shorten (lengthen) the product life; on the other hand, (ii) a shorter (longer) product life will in turn result in a lower (higher) disposal fraction. In this sense, the overall impacts are far away from simple and straightforward, as presented in the following two propositions.

Proposition 4.4   *Given* $0 < \mu < \rho$, $\frac{\partial Q}{\partial S} < 0$ *if* $s < s^*$; $\frac{\partial Q}{\partial S} > 0$ *if* $s > s^*$ *where*

$$s^* = WL[\gamma(\bar{C} - C_0) + d_0] + \bar{C} + \frac{\alpha(C_0 - \bar{C})}{\rho - \mu} + \frac{[\alpha - (\rho - \mu)(1 + \gamma WL)](\bar{C}^{\beta_1} - C_0^{\beta_1})}{(\rho - \mu)\beta_1 \bar{C}^{\beta_1 - 1}},$$ *and* $\bar{c}$ *is the solution of*

$\gamma C(\ln C - \ln C_0) - \gamma(C - C_0) - d_0 = 0$.

See Appendix N for proof. This proposition indicates that, if the remanufacturing subsidy is relatively low (high), an increase of the subsidy will result in a lower (higher) level of the dispos-

al rate. Also, S^* represents the optimal remanufacturing subsidy that will lead to the minimum disposal rate.

The intuitive interpretation of this proposition is as follows. A high remanufacturing subsidy implies a low remanufacturing cost threshold (Proposition 4.3), which in turn results in a low disposal fraction (Assumption 4.8). In this case, the major constituents of the disposed materials come from components that are replaced regardless of the operating life such as plastic covers in copiers (i.e., term d_0 in Assumption 4.8). Therefore, the increase of the subsidy becomes less helpful in reducing the disposal fraction. Meanwhile, a higher subsidy does result in a shorter product life (i.e., more frequent disposal). Therefore, in totality, when increases the subsidy at a high level, the increase of the disposal frequency dominates the decrease of disposal fraction, which results in a higher disposal rate (i.e., weight of disposed materials per unit time). Therefore, the increase of the remanufacturing subsidy does not always lead to less environmental impact, and hence, should not be advocated unconditionally.

Proposition 4.5 *Given* $0 < \mu < \rho$, $\frac{\partial Q}{\partial L} < 0$ *if* $L < L^*$; $\frac{\partial Q}{\partial L} > 0$ *if* $L > L^*$ *where*

$$L^* = \frac{(\alpha - \rho + \mu)[(\beta_1 - 1)\bar{C}^{\beta_1} + C_0^{\beta_1}] - [\alpha C_0 - S(\rho - \mu)]\beta_1 \bar{C}^{\beta_1 - 1}}{\gamma W(\rho - \mu)[\beta_1 \bar{C}^{\beta_1}(\ln \bar{C} - \ln C_0) - (\bar{C}^{\beta_1} - C_0^{\beta_1})]}, \text{ and } \bar{C} \text{ is the solution of}$$

$$\gamma C(\ln C - \ln C_0) - \gamma(C - C_0) - d_0 = 0.$$

See Appendix O for proof. This proposition indicates that, if the disposal fee is relatively low (high), an increase of the disposal fee will result in a lower (higher) disposal rate. Also, L^* represents the optimal disposal fee that will lead to the minimum disposal rate.

Intuitively, a high disposal fee implies a long product life (Proposition 4.3), which in turn implies a high maintenance cost (Assumption 4.4). Therefore, due the cumulative impact of

maintenance cost on project value, the increase of disposal fee becomes less helpful in increasing the product life (i.e., lower disposal frequency). Meanwhile, a higher disposal fee does result in a higher disposal fraction (Proposition 4.3 and Assumption 4.8). Therefore, in totality, when increases the disposal fee at a high level, the increase of the disposal fraction dominates the decrease of the disposal frequency, which results in a higher disposal rate. This ramification implies that increasing disposal fee may not always prevent wastes from being dumped at landfill, and should be implemented more carefully.

Proposition 4.6   *Given* $0 < \mu < \rho$, $\dfrac{\partial S^*}{\partial \sigma} > 0$ *and* $\dfrac{\partial L^*}{\partial \sigma} < 0$.

See Appendix P for proof. This proposition indicates that, as the remanufacturing cost volatility increases, the optimal remanufacturing subsidy increases and the optimal disposal fee decreases. Hence, when observing higher volatility in remanufacturing cost from industries, policy makers are advised to either increase the remanufacturing subsidy or decrease the disposal fee so as to maintain the minimum environmental impact. The intuition behind this proposition is as follows. Since a higher volatility in remanufacturing cost incentivizes the decision maker to defer the remanufacturing option, the policy maker needs to adjust the subsidy/fee reversely so as to balance the deference caused by the higher volatility. Therefore, by Proposition 4.3, the policy maker should reduce the exercising cost of the option by either increasing the remanufacturing subsidy or decreasing the disposal fee. This proposition provides a guideline for policy makers regarding how economic instruments should be adjusted in response to the volatility change in remanufacturing industries.

4.3   Numerical Study

In this section, utilizing the remanufacturing of photocopier as an example, we numerically illustrate the key features of our analytical findings in the previous section. Photocopier remanufacturing is one of the most mature and widely implemented remanufacturing processes in practice. Xerox has developed a remanufacturing system since late 1980s to maximize the profitability of operations, and saved millions of dollars in raw materials and disposal costs [33]. At the same time, photocopier is also widely utilized by firms such as Staples to provide services to customers. Now, let us first present the parameter values used in this study. Even though the values are hypothetical, we have consulted the local Staples Managers as well as others [16]-[18]. These values are summarized in Table 1, and the corresponding numerical results are given in Table 2.

Table 1. Parameters and Values

Parameters	Values
Leasing Price P ($/year) (see Assumption 4.2)	*800*
Cleaning, Testing, and Disassembly Cost C_0 ($/unit product) (see Assumption 4.3)	*250*
Annual Discount Rate ρ	*0.10*
Annualized Growth Rate of Remanufacturing Cost μ (see Assumption 4.3)	*0.08*
Annualized Volatility of Remanufacturing Cost σ (see Assumption 4.3)	*0.04*
Product Weight W (lb) (see Assumption 4.7)	*300*
Remanufacturing Subsidy S ($/unit product) (see Assumption 4.6)	*30*
Disposal Fee L ($/lb) (see Assumption 4.7)	*0.03*
Maintenance Cost Function Coefficient α (see Assumption 4.4)	*0.3*
Disposal Fraction Function Coefficient γ (%/$) (see Assumption 4.8)	*0.5*
Disposal Fraction Function Coefficient d_0 (see Assumption 4.8)	*10%*

Table 2. Numerical Results Relevant to the Remanufacturing Decision

Variables	Values
A_1	8.98
β_1	1.25
Remanufacturing Cost Threshold C_R ($/unit product)	361.93
Product Life (year)	4.67
Disposal Fraction $d(C)$	65.97%
Disposal Rate Q (lb/year)	41.93
Optimal Remanufacturing Subsidy S ($/unit product)	44.47
Optimal Disposal Fee L ($/lb)	0.04
Project Value at the beginning of the Product Life $V(C_0)$ ($)	16,435
Project Value at the end of the Product Life $V(C_R)$ ($)	16,073
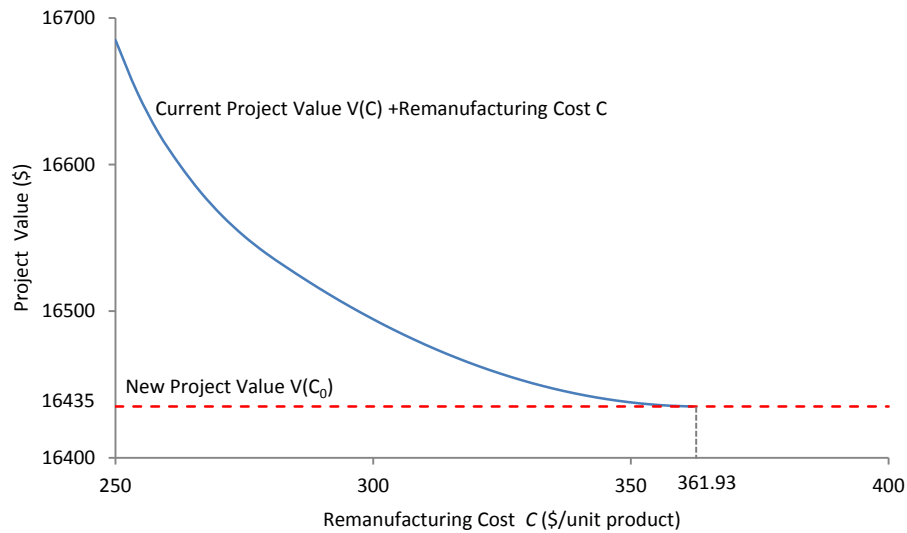
Figure 6. Project Value vs. Remanufacturing Cost

Figure 6 shows the value of the project with respect to the remanufacturing cost. It can be observed that, as the remanufacturing cost increases, the currently operating project value decreases. When the sum of the current project value and the remanufacturing cost (blue solid

line) equals to the new project value (red dashed line), it is optimal to remanufacture the product. The intersection of the blue line and the red line characterizes the value-matching and smooth-pasting condition at the remanufacturing cost threshold. Figure 7 shows the variation of the remanufacturing cost threshold with the remanufacturing cost volatility and the fixed cost (e.g., cleaning, testing, and disassembly). The numerical results are consistent with the analytical results given in Proposition 4.2.
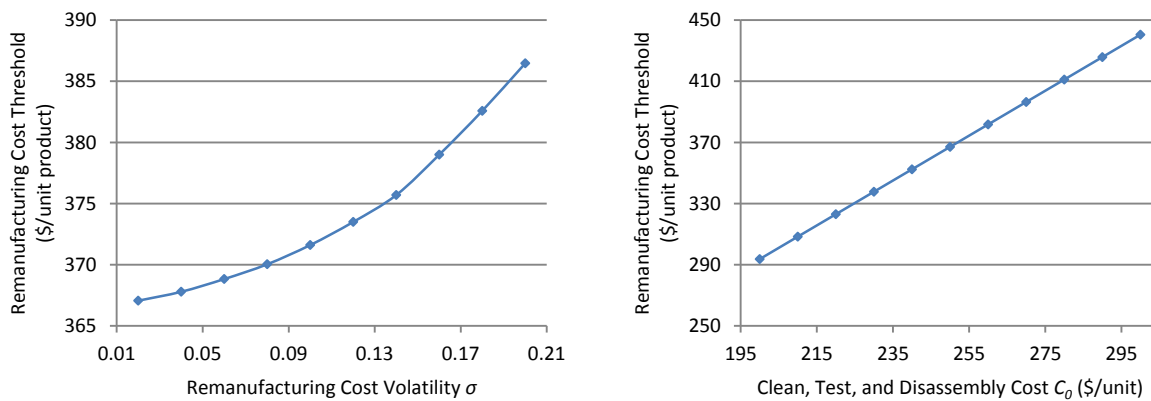
Figure 7. Variation of Remanufacturing Cost Threshold with Volatility and Fixed Cost

Figure 8 shows the variation of the disposal rate with the remanufacturing subsidy and the disposal fee, which is also consistent with the analytical results presented in Proposition 4.4 & 4.5. Moreover, Figure 9 shows the variation of the optimal remanufacturing subsidy and the optimal disposal fee with the remanufacturing cost volatility, and the numerical results align with the analytical conclusion presented in Proposition 4.6.
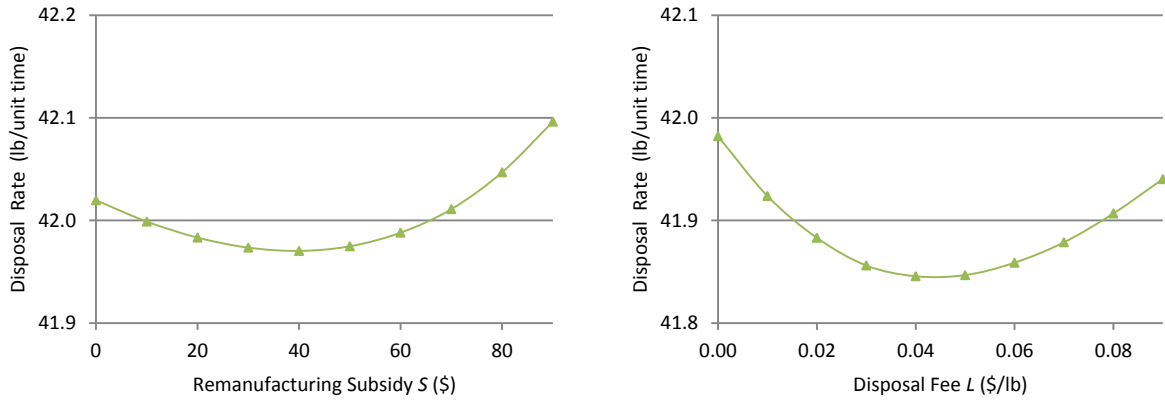
Figure 8. Variation of Reuse Fraction, Product Life, and Disposal Rate with S and L
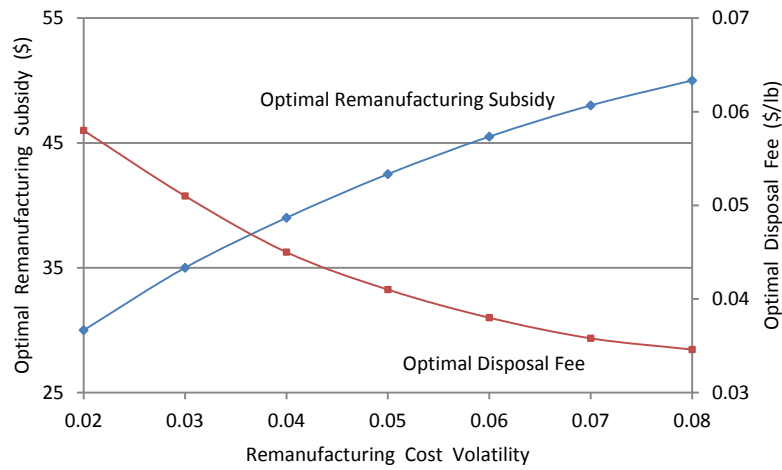
Figure 9. Variation of Optimal Remanufacturing Subsidy and Disposal Fee with Volatility

Thus far, we have performed the numerical analysis for the firm's remanufacturing decision based on fixed hypothetical data. Now, in contrast, Monte Carlo method is utilized to simulate the sample paths given $c_0 = 250$ (\$/unit) as the starting point and the GBM process as $dC = 0.08Cdt + 0.04Cdz$.

Given the parameters as indicated before, the GBM paths were generated 10,000 times by MATLAB, and the mean product life was 4.89 years with 95% confidence interval (4.64, 5.14), which is fairly close to the expected product life of 4.67 years obtained earlier this section. Here the confidence interval comes from standard procedure like [84][85][89]. Furthermore, two

typical sample paths with same growth rate (0.08) and different volatility (0.04 and 0.06) are given in Figure 10 (sample path with volatility 0.04 in blue and sample path with volatility 0.06 in red). It can be observed that, the remanufacturing threshold increases with a higher volatility, which induces decision makers to hold the option for a longer time. Intuitively, after reaching $361.93 (i.e., the remanufacturing threshold in the low-volatility case) for the first time around the 5th year, the high-volatility sample path falls below $361.93 several times. Hence, with a higher volatility, it is advised to defer the exercise of the remanufacturing option. This observation aligns with our previous statement that, with a higher volatility, there is a greater chance of a reduction in the remanufacturing cost in the near future.
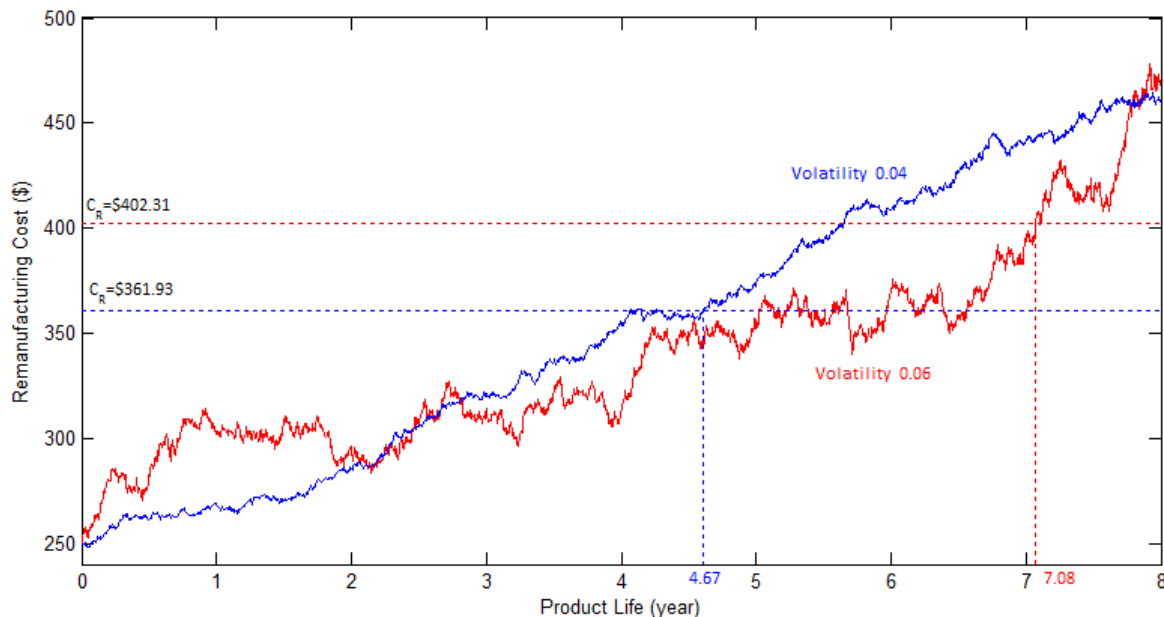
Figure 10. Sample Paths of the Remanufacturing Cost in 8 Years

4.4   Managerial Insights and Policy Implications

A higher remanufacturing cost volatility entails a higher remanufacturing cost threshold. This finding implies that the remanufacturing cost uncertainty is the driver for the decision maker to defer the exercise of the remanufacturing option. Therefore, any factor that will cause uncer-

tainty in remanufacturing cost directly or indirectly should be carefully examined. For example, if the market prices of the product components tend to fluctuate more intensively, or the service demand becomes more unpredictable (resulting in uncertainty in operating frequency and component deterioration accordingly), it is advised that the decision maker should wait for a higher remanufacturing cost threshold to exercise the remanufacturing option. Furthermore, if there is any external incentive (penalty) for product remanufacturing, exercising the remanufacturing option at a lower remanufacturing cost is recommended.

From government's perspective, introducing a remanufacturing subsidy will incentivize the industrial decision maker to remanufacture products more frequently at the cost of a shorter product life. On the other hand, imposing a higher disposal fee incentivizes the decision maker to prolong the product life at the cost of a higher disposal fraction of end-of-life products. The analytical results culminated in Proposition 4.4 and 4.5 imply that, counter-intuitively, increasing the remanufacturing subsidy may result in more raw materials consumptions while increasing the disposal fee may result in more wastes being dumped at landfills. As shown in the last two graphs in Figure 8, the disposal rate increases with the remanufacturing subsidy and the disposal fee after the subsidy reaches \$44.47 and the fee reaches \$0.04/lb. For instance, the wisdom that public landfills (operated by local community) can convert wastes to other environmental-friendly alternatives by charging a higher tipping fee is widely observed in the literature and/or government reports [32]. However, this study demonstrated that the ramifications resulted from a higher disposal cost may actually harm the environment. Therefore, any form of remanufacturing subsidy or disposal penalty should be carefully examined before implementation.

Furthermore, considering the less than stable economic conditions of nowadays, it is desirable to provide guidelines for the government regarding how policies should be adjusted timely and properly in response to the uncertainties. In this regard, Proposition 4.6 provides such a guideline for policy makers to adjust the remanufacturing subsidy and disposal fee when the remanufacturing cost volatility varies. Such a guideline, we think, will be vital as the economic conditions become even more uncertain.

CHAPTER 5

REMANUFACTURING AND REPLACEMENT DECISIONS UNDER

OPERATIONS AND MAINTENANCE COST UNCERTAINTY

5.1   Background and Objectives

In recent years, there have been much interests in remanufacturing because of its focus on sustainability with often positive environmental as well as economic impacts [11]. During the remanufacturing process, a product is recovered at a part level with worn-out parts being replaced by new ones and durable parts being reused again. For example, in photocopier remanufacturing, mechanical parts such as bushings are replaced by new ones and electrical modules such as scanning motor are reused [47]. In single-use camera remanufacturing, plastic covers are replaced while printed circuit boards and lenses are reused.

In current industrial practice, the key driver of the remanufacturing process is the economic value of the reused parts. Overall, remanufacturing programs can save companies 40%–65% of the manufacturing cost [1]. In particular, Xerox's remanufacturing program saved $200 million in material and part cost in less than five years [5]. Given these economic benefits, substantial efforts have been made by practitioners from a variety of industries to retrieve the potential value in the used parts by improving their durability. Thanks to these efforts, from the product design perspective, photocopier can be remanufactured up to 6 times [63]; single-use camera can be remanufactured up to 5 times [64]; tire casings and computer chips can be reused up to 3 and 4 times, respectively [5]. However, Ayres and Ayres [65] claimed that the appropriate

number of times a product should be remanufactured before being replaced is essentially driven by the economic factors such as operations and maintenance (O&M) costs.

In this paper, we further develop the research question in the line of Ayres and Ayres [65]. Namely, the aim of this paper is to answer what the optimal remanufacturing and replacement policies are for the non-durable parts and for the product. This type of study is critical for the maintenance and replacement research as one could view the remanufacturing as a very sustainable means of a maintenance process. At this point in time, however, to our knowledge, there are few quantitative studies addressing this critical issue in the literature – especially when the O&M costs have uncertainties. We note that, in this paper, we will use the terms policies and decisions interchangeably.

Toward this goal, we assume that a product consists of durable and non-durable parts with uncertain, yet different, O&M costs. Moreover, to aim for more concrete analyses and intuitions, we consider a firm that leases a product to a service provider, and the firm is assumed to be responsible for the O&M costs of the product. The O&M costs for the durable parts and the non-durable parts are assumed to follow uncorrelated geometric Brownian motion (GBM) processes. Throughout the product's life-cycle, the firm has two options for the product: remanufacturing option and replacement option. With these options, the firm has the right, but not the obligation, to remanufacture or replace the product at any point. In particular, the exercise of the remanufacturing option triggers the ultimate maintenance for the product by replacing the non-durable parts with new ones. The replacement option triggers the replacement of the product as a whole. Under this framework, this study attempts to

(1) Formulate and analyze model of product remanufacturing and replacement decisions from a real options perspective when the O&M costs for both durable parts (reused in remanu-facturing process) and non-durable parts (replaced in remanufacturing process) follow GBM processes.

(2) Investigate the optimal timing to remanufacture and replace a product under uncertain O&M costs, and derive the appropriate number of times that the product should be re-manufactured.

(3) Examine how the O&M cost uncertainties and other relevant factors influence the firm's remanufacturing and replacement decisions.

(4) Evaluate the efficiency of the governmental remanufacturing subsidy in reducing the land-fill disposal, and provide critical guidelines for the governmental sustainable policies.

Furthermore, the analytical and numerical findings include that (i) as the O&M costs become more (less) volatile, it is beneficial to exercise the remanufacturing and the replacement op-tions later (earlier); (ii) a higher (lower) remanufacturing cost results in a shorter (longer) ex-pected product life because it becomes less attractive for the firm to conduct ultimate mainte-nance for the product through remanufacturing; (iii) an increase of the remanufacturing subsidy may result in more landfill disposal per unit time, and the government policy maker is advised to carefully examine the industrial conditions before any implementation of a new sustainable subsidy or adjustment of the existing sustainable subsidy. We hope that the questions raised and addressed in this study will stimulate relevant discussions among industrial practitioners, governmental regulators, and academics.

5.2   Model

In this section, let us consider a firm that leases a product to a service provider and the service provider utilizes the product to serve customers. Meanwhile, the firm is responsible for the O&M costs. Such leasing contracts exist in a variety of industries, such as photocopier leasing contract between Xerox and Staples and mail processing machine leasing contract between Pitney Bowes and USPS. Furthermore, at any time point, the firm has an option to remanufacture the product by replacing the non-durable parts, or replace the whole product by a new one. The exercise of these options is driven by the O&M cost of the durable parts which deteriorate over the product's life cycle, and the O&M cost of the non-durable parts which are replaced after each remanufacturing process. Prior to investigating the optimal timing of product remanufacturing and replacement, we first provide the key assumptions of our model.

Assumption 5.1   *There exists a leasing contract between the firm and the service provider specifying the leasing price per unit product P ($/unit product/unit time). Furthermore, the contract is sufficiently long that the expiration of the contract is not considered in the model.*

Assumption 5.2   *The durable-part O&M cost C_D ($/unit time) and non-durable-part O&M cost C_N ($/unit time) follow two uncorrelated GBM processes. Specifically,*

$$dC_D = \mu_D C_D dt + \sigma_D C_D dz_D \tag{5.1}$$
$$dC_N = \mu_N C_N dt + \sigma_N C_N dz_N \tag{5.2}$$

Here μ_D and μ_N represent the instantaneous drift; σ_D and σ_N represent the instantaneous volatility; dz_D and dz_N are two standard Brownian motion increments. Moreover, the starting O&M cost for the durable parts (when the product is new) and the starting O&M cost for the non-durable parts (right after remanufacturing) are c_{D0} and c_{N0}, respectively.

The volatilities in these two GBM processes are the sources of uncertainties in our remanufacturing and replacement model. As in the case of photocopiers, the durable-part O&M cost includes the costs associated with sensors cleaning, chip replacement, etc. Furthermore, as a photocopier ages and deteriorates, the fuser assembly may overheat and cause severe damage to the motor, which could cost over a hundred dollars to recover [66]. Meanwhile, the non-durable-part O&M cost includes the costs associated with bearings re-oil, bushings replacement, etc. The volatility of the O&M costs could result from the uncertain frequency of cleaning, replacement, and re-oil process. Moreover, modeling O&M cost as a GBM cost can be observed in the literature [67][68].

The justification for the uncorrelated GBM processes is given as follows. In recent years, the increase of the modularized design facilitates the maintenance of the products and results in more independently O&M costs for parts and modules [69]. For instance, the maintenance for the scanning motor is independent of the maintenance for the bearings and bushings in the photocopiers [66]. Furthermore, the assumption of uncorrelated GBM processes facilitates the mathematical tractability of our model, and the case of correlated GBM processes is beyond the scope of this study, which could serve as an interesting future extension.

Assumption 5.3   *The remanufacturing cost for a used product is K_R ($/unit product); the manufacturing cost for a new product is K_A ($/unit product) where $K_R < K_A$. The period to manufacture a new product or remanufacture a used product is negligible.*

A typical remanufacturing process includes cleaning, testing, disassembling, component replacement, and assembling processes. Here $K_R < K_A$ implies a cost saving in the remanufacturing process due to the economic value of reused parts, which is the key driver of product re-

manufacturing [13]. This simplifying assumption of negligible manufacturing/remanufacturing period is made to focus on the remanufacturing and replacement decisions without diluting our attention to various transient issues such as how to make best decisions during the manufacturing/remanufacturing process.

5.2.1 Remanufacturing Decision

Under the assumptions and modeling framework presented above, the remanufacturing decision can be interpreted as an optimal stopping problem. Specifically, since the durable parts are not replaced during the remanufacturing process, at any time point, the decision maker measures the non-durable-part O&M cost and the project value to decide whether or not to exercise the remanufacturing option. The project value is defined as the sum of the discounted cash flow resulted from the product and the value of remanufacturing option. Due to the nature of the optimal-stopping problem, there exists a threshold of the non-durable-part O&M cost above which it is optimal to remanufacture the product. Furthermore, before the exercise of the remanufacturing option, the project value $V(C_D,C_N)$ must satisfy the following Bellman's equation [59].

$$\rho V dt = (P - C_D - C_N) dt + E[dV | C_D, C_N] \tag{5.3}$$

where ρ is the discount rate per unit time, and $E[]$ the operator of expected value. The left-hand side of (5.3) is the return per unit time for holding the remanufacturing option. On the right-hand side, the first term is the immediate profit from the product, and the second term is the expected depreciation of the product value, which is conditioned on the instantaneous O&M costs.

Furthermore, by Ito's lemma, the following equation can be derived

$$dV = \frac{dV}{dC_D}dC_D + \frac{dV}{dC_N}dC_N + \frac{1}{2}\frac{d^2V}{dC_D^2}(dC_D)^2 + \frac{1}{2}\frac{d^2V}{dC_N^2}(dC_N)^2 \tag{5.4}$$

Substituting eq. (5.3) into eq. (5.4), the following equation can be derived

$$dV = \frac{dV}{dC_D}\mu_D C_D dt + \frac{dV}{dC_N}\mu_N C_N dt + \frac{1}{2}\frac{d^2V}{dC_D^2}\sigma_D^2 C_D^2 dt + \frac{1}{2}\frac{d^2V}{dC_N^2}\sigma_N^2 C_N^2 dt \tag{5.5}$$

To guarantee the convergence of the differential equation (5.5), we assume $\mu_D > \rho$ and $\mu_N > \rho$ (Costa Lima and Suslick, 2006). It can be verified that the product value function is (see Appendix Q for proof)

$$V(C_D, C_N) = A_D C_D^{\beta_D} + A_N C_N^{\beta_N} - \frac{C_D}{\rho - \mu_D} - \frac{C_N}{\rho - \mu_N} + \frac{P}{\rho} \tag{5.6}$$

where $\beta_D = \left[\frac{1}{2}\sigma_D^2 - \mu_D + \sqrt{\left(\frac{1}{2}\sigma_D^2 - \mu_D\right)^2 + 2\rho\sigma_D^2}\right] \Big/ \sigma_D^2 > 1$, $\beta_N = \left[\frac{1}{2}\sigma_N^2 - \mu_N + \sqrt{\left(\frac{1}{2}\sigma_N^2 - \mu_N\right)^2 + 2\rho\sigma_N^2}\right] \Big/ \sigma_N^2 > 1$.

Suppose C_{NR} is the threshold of non-durable-part O&M cost above which it is optimal to exercise the remanufacturing option, then the following boundary conditions must be satisfied.

$$V(C_D, C_{NR}) = V(C_D, C_{N0}) - K_R \quad \Leftrightarrow \quad A_N C_{NR}^{\beta_N} - \frac{C_{NR}}{\rho - \mu_N} = A_N C_{N0}^{\beta_N} - \frac{C_{N0}}{\rho - \mu_N} - K_R \tag{5.7}$$

$$\frac{\partial V(C_D, C_{NR})}{\partial C_N} = \frac{\partial [V(C_D, C_{N0}) - K_R]}{\partial C_N} \quad \Leftrightarrow \quad A_N \beta_N C_{NR}^{\beta_N - 1} - \frac{1}{\rho - \mu_N} = 0 \tag{5.8}$$

Equation (5.7) is the value-matching condition, which guarantees that, at the threshold C_{NR}, the current project value equals to the project value after remanufacturing (i.e., the non-durable part is like new) minus the remanufacturing cost. Meanwhile, the smooth-pasting conditions assure the continuity and smoothness at the exercising point.

Proposition 5.1 *Given $0 < \mu_N < \rho$, the threshold of non-durable-part O&M cost for the remanufacturing option c_{NR} is the solution of $(\beta_N - 1)c_{NR}^{\beta_N} - [K_R(\rho - \mu_N) + C_{N0}]\beta_N c_{NR}^{\beta_N - 1} + c_{N0}^{\beta_N} = 0$.*

Proof is given in Appendix R, and the value of C_{NR} can be solved by computational software (e.g., Matlab). As soon as the non-durable-part O&M cost is observed to be not less than C_{NR}, it is beneficial for the firm to remanufacture the product. The remanufacturing process reduces the non-durable-part O&M cost to the original level, which implies a higher project value. Furthermore, the remanufacturing decision is irrelevant to the durable-part O&M cost as this O&M cost is not influenced by the remanufacturing process.

Proposition 5.2   *Given* $0 < \mu_N < \rho$, $\dfrac{\partial C_{NR}}{\partial \sigma_N} > 0$, $\dfrac{\partial C_{NR}}{\partial K_R} > 0$, *and* $\dfrac{\partial C_{NR}}{\partial C_{N0}} > 0$.

Proof is given in Appendix R. This proposition indicates that, as the non-durable-part O&M cost becomes more volatile, the remanufacturing option threshold increases, which implies that the exercise of remanufacturing option should be deferred. This is because, as the non-durable-part O&M cost volatility increases, the flexibility to remanufacture the product at any time point (i.e., remanufacturing option) becomes more valuable, and it is beneficial to hold the option for longer. Moreover, the increase of the remanufacturing cost results in a higher remanufacturing option threshold. Intuitively, a higher remanufacturing cost K_R implies a higher penalty to exercise the option, which incentivizes the firm to defer the exercise of the remanufacturing option. Meanwhile, a higher level of the original O&M cost for the non-durable parts implies a lower benefit of exercising the remanufacturing option, and hence, incentivizes the firm to exercise the remanufacturing option later.

Thus far, we have performed analysis against the non-durable-part O&M cost threshold for the remanufacturing option. We now proceed to study the expected life of the non-durable parts.

Proposition 5.3 *Given* $\mu_N > \sigma_N^2/2$, *the expected life of non-durable parts* $T_N = \dfrac{\ln C_{NR} - \ln C_{N0}}{\mu_N - \sigma_N^2/2}$.

Corollary 5.1 *Given* $\mu_N > \sigma_N^2/2$, $\dfrac{\partial T_N}{\partial \sigma_N} > 0$, $\dfrac{\partial T_N}{\partial K_R} > 0$, *and* $\dfrac{\partial T_N}{\partial C_{N0}} < 0$.

The proof of Proposition 5.3 and Corollary 5.1 is given in Appendix S. The fact that the expected life of non-durable parts increases with the O&M cost volatility and the remanufacturing cost is intuitive. As for $\partial T_N / \partial C_{N0} < 0$, even though a higher level of the starting non-durable-part O&M cost C_{N0} results in a higher threshold of the O&M cost (Proposition 5.2), it also sets a higher starting point of the non-durable-part O&M cost. Therefore, the overall impact of C_{N0} is not simple and straightforward. This corollary indicates that the expected life of non-durable parts decreases with the starting O&M cost for the non-durable parts C_{N0}.

5.2.2 Replacement Decision

Suppose C_{DA} and C_{NA} are the durable-part O&M cost and the non-durable-part O&M cost when the replacement option is exercised, respectively. Hence, the value-matching and smooth-pasting conditions at the replacement point become

$$V(C_{DA},C_{NA}) = V(C_{D0},C_{N0}) - K_N \Leftrightarrow A_D C_{DA}^{\beta_D} + A_N C_{NA}^{\beta_N} - \frac{C_{DA}}{\rho - \mu_D} - \frac{C_{NA}}{\rho - \mu_N} = A_D C_{D0}^{\beta_D} + A_N C_{N0}^{\beta_N} - \frac{C_{D0}}{\rho - \mu_D} - \frac{C_{N0}}{\rho - \mu_N} - K_A \quad (5.10)$$

$$\frac{\partial V(C_{DA},C_N)}{\partial C_D} = \frac{\partial V(C_{D0},C_{N0}) - K_N}{\partial C_D} \Rightarrow A_D \beta_D C_{DA}^{\beta_D - 1} - \frac{1}{\rho - \mu_D} = 0 \quad (5.11)$$

$$\frac{\partial V(C_D,C_{NA})}{\partial C_N} = \frac{\partial V(C_D,C_{N0}) - K_N}{\partial C_N} \Rightarrow A_N \beta_N C_{NA}^{\beta_N - 1} - \frac{1}{\rho - \mu_N} = 0 \quad (5.12)$$

Proposition 5.4 *Given* $0 < \mu_D, \mu_N < \rho$, *it is optimal to exercise the replacement option if*

$F(C_D,C_N) \geq K_A$ *where* $F(C_D,C_N) = \dfrac{C_D - C_{D0}}{\rho - \mu_D} + \dfrac{C_N - C_{N0}}{\rho - \mu_N} - \dfrac{C_D^{\beta_D} - C_{D0}^{\beta_D}}{(\rho - \mu_D)\beta_D C_D^{\beta_D - 1}} - \dfrac{C_N^{\beta_N} - C_{N0}^{\beta_N}}{(\rho - \mu_N)\beta_N C_N^{\beta_N - 1}}$.

Proof is given in Appendix T. This proposition provides the threshold condition for the replacement option. An any time, by observing the O&M cost for durable and non-durable parts, the firm measures the value of $F(C_D,C_N)$ and decides whether or not to retire and replace the

product. $F(C_D, C_N)$ is defined as the value-difference function, which measures the difference between the new project value (after replacement) and the current project value. Intuitively, as soon as the value-difference equals to the manufacturing cost, it is optimal to retire the current product and replace it with a new one.

Furthermore, we note that, due to the nature of GBM process, at the replacement point, the value of C_{DA} and C_{NA} may (if not must) be different case by case, depending on how both O&M costs evolve with respect to time. Hence, there is no individual threshold for the durable-part O&M cost and the non-durable-part O&M cost for the replacement option.

Corollary 2: $Given$ $0 < \mu_D, \mu_N < \rho$, $\frac{\partial F(C_D, C_N)}{\partial C_D} > 0$ and $\frac{\partial F(C_D, C_N)}{\partial C_N} > 0$.

See Appendix T for proof. This corollary provides an alternative interpretation for Proposition 5.3. That is, given a fixed non-durable-part (durable-part) O&M cost, there exists a threshold of the durable-part (non-durable-part) O&M cost above which it is optimal to exercise the replacement option.

Proposition 5.5   *Given a fixed non-durable-part O&M cost,* $\frac{\partial C_{DA}}{\partial \sigma_D} > 0$, $\frac{\partial C_{DA}}{\partial \sigma_N} > 0$, $\frac{\partial C_{DA}}{\partial K_A} > 0$; *given a fixed durable-part O&M cost,* $\frac{\partial C_{NA}}{\partial \sigma_D} > 0$, $\frac{\partial C_{NA}}{\partial \sigma_N} > 0$, $\frac{\partial C_{NA}}{\partial K_A} > 0$.

Proof is given in Appendix U. As the durable-part or the non-durable-part O&M cost becomes more volatile, given a fixed non-durable-part (durable-part) O&M cost, the corresponding durable-part (non-durable-part) O&M cost threshold for the replacement option becomes higher. In other words, a higher O&M cost volatility incentivizes the firm to hold the product for longer, and defer the exercise of the replacement option. This is because a higher O&M cost volatility implies a higher value of the replacement option. Similarly, a higher manufacturing

cost also implies a deferred exercise of the replacement option, which leads to a longer product life.

Lemma 1   *Given* $0 < \mu_D, \mu_N < \rho$, $\dfrac{dC_{DA}}{dC_{NA}} < 0$.

Proof is given in Appendix V. According to Proposition 5.1, throughout the life-cycle of the product, the maximum non-durable-part O&M cost is C_{NR}, at which the product is remanufactured and the non-durable-part O&M cost is restored to the original level of C_{N0}. Therefore, by this lemma, the minimum durable-part O&M cost that can trigger the exercise of replacement option can be derived as presented in the following proposition.

Proposition 5.6   *The lower bound of the durable-part O&M cost to exercise the replacement option is $\bar{c}_{DA}$ where $\bar{c}_{DA}$ is the solution of $(\beta_D - 1)\bar{c}_{DA}^{\beta_D} - [(K_A - K_R)(\rho - \mu_D) + C_{D0}]\beta_D \bar{c}_{DA}^{\beta_D - 1} + C_{D0}^{\beta_D} = 0$.*

Proof is given in Appendix V. This proposition implies that a product will not be retired and replaced until the durable-part O&M cost is greater than $\bar{c}_{DA}$. Also, we want emphasize that, different from the concept of threshold, it may not be optimal to exercise the replacement option even if the durable-part O&M cost is beyond this lower bound. The actual durable-part O&M cost triggering the exercise of the replacement option depends on the instantaneous non-durable-part O&M cost. By this proposition, the lower bound of the expected life of the product can also be derived as $T = (\ln \bar{c}_{DA} - \ln C_{D0})/(\mu_D - \sigma_D^2/2)$.

Corollary 5.3 *Given* $0 < \mu_D < \rho$, $\dfrac{d\bar{c}_{DA}}{dK_R} < 0$.

Proof is given in Appendix W. This corollary indicates that, as the remanufacturing cost increases, the lower bound of the durable-part O&M cost to replace the product decreases. Intuitively, as product remanufacturing becomes more expensive, the alternative option of replacing

the product becomes more attractive. Therefore, the condition to exercise the replacement option becomes less restricted (i.e., the lower bound $\bar{c}_{DA}$ decreases).

Thus far, a series of critical analytical results have been presented. In what follows, we will perform an extensive numerical study to further illustrate these findings and gain more observations.

5.3 Numerical Study

In this section, we utilize the remanufacturing and replacement decisions of photocopiers as the study case to conduct the numerical analysis. Photocopiers are widely leased by manufacturers such as Xerox and Fujifilm to service provider such as Staples and Office Depot to provide copying, printing, and scanning services to the customers. Meanwhile, photocopier remanufacturing is one of the most mature and widely implemented remanufacturing processes. Xerox has developed a remanufacturing system since late 1980s to maximize the profitability of operations, and has saved millions of dollars in raw materials and disposal costs (Kerr and Ryan, 2001). Let us first present the parameter values used in this study. Even though some of the parameter values are hypothetical due to the lack of the empirical data, we have consulted local Staples managers as well as others (Fuji Xerox, 2002; Geyer et al., 2007). The values are summarized in Table 1, and some of the key numerical results are given in Table 2.

Table 3. Numerical Study Parameter Values

Parameters	Symbol	Values
Monthly discount rate (%/month)	ρ	*0.10*
Leasing price per unit product ($/month, Assumption 5.1)	P	*150*
Original durable-part O&M cost ($/month, Assumption 5.2)	c_{D0}	*25*
Original non-durable-part O&M cost ($/month, Assumption 5.2)	c_{N0}	*50*

Drift of durable-part O&M cost (Assumption 5.2)	μ_D	*0.06*
Drift of non-durable-part O&M cost (Assumption 5.2)	μ_N	*0.08*
Volatility of durable-part O&M cost (Assumption 5.2)	σ_D	*0.03*
Volatility of non-durable-part O&M cost (Assumption 5.2)	σ_N	*0.04*
Remanufacturing cost per unit product ($/unit product, Assumption 5.3)	κ_R	*350*
Manufacturing cost per unit product ($/unit product, Assumption 5.3)	κ_A	*600*

Table 4. Numerical Results

Decisions	Solutions
Non-durable-part O&M cost threshold for remanufacturing option c_{NR}	*111.24 ($/month)*
Expected life of non-durable part τ_N	*10.10 (month)*
Lower bound of durable-part O&M cost for replacement option $\bar{c}_{DA}$	*67.46 ($/month)*
Lower bound of expected life of product (month)	*16.67 (month)*

 Furthermore, Figure 11 shows the variation of C_{NR} (i.e., the non-durable-part O&M cost threshold for remanufacturing option) and T_N (i.e., the expected non-durable-part life) with σ_N (i.e., the non-durable-part O&M cost volatility), K_R (i.e., the remanufacturing cost), and C_{N0} (i.e., the original non-durable-part O&M cost). If a parameter value is not changing in the sensitivity analysis, it takes the value in Table 4. The numerical results in Figure 11 align with the analytical findings given in Proposition 5.2 and corollary 5.1.
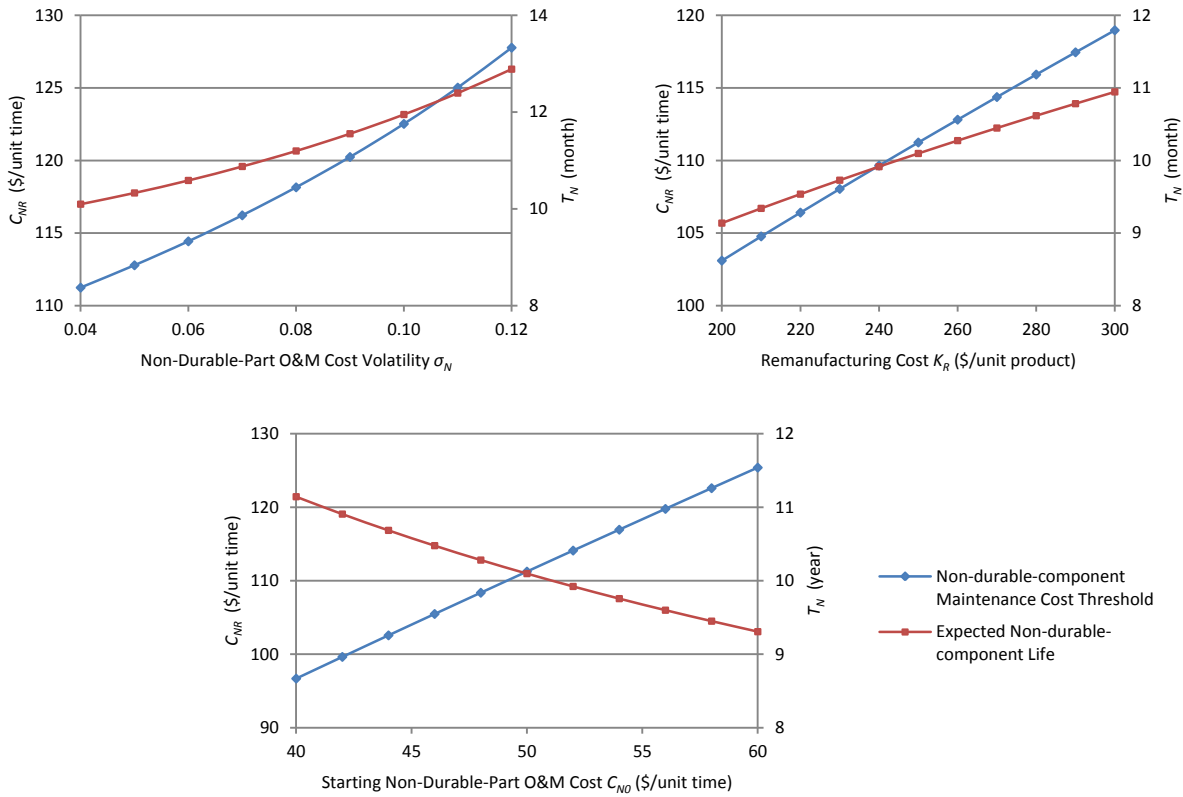
Figure 11. Variation of C_{NR} and T_N with Volatility, Remanufacturing Cost, and Starting O&M Cost

Given the parameter values listed in Table 1, the GBM process of the durable-part O&M cost and the non-durable-part O&M cost become $dC_N = 0.08C_N dt + 0.04C_N dz_N$ and $dC_D = 0.06C_D dt + 0.03C_D dz_D$, respectively. Also, the project-value-difference function $F(C_D, C_N)$ becomes

$$F(C_D,C_N) = 4.51C_D + 19.88C_N + 1040.16C_D^{-0.22} + 19913.86C_N^{-0.66} - 3125 \qquad (5.13)$$

By plugging in the instantaneous durable-part O&M cost and non-durable-part O&M cost in-to (5.13), the decision make can compare the value of $F(C_D,C_N)$ to the manufacturing cost $K_A = 600$ (\$/unit product) and decide whether or not to exercise the replacement option.
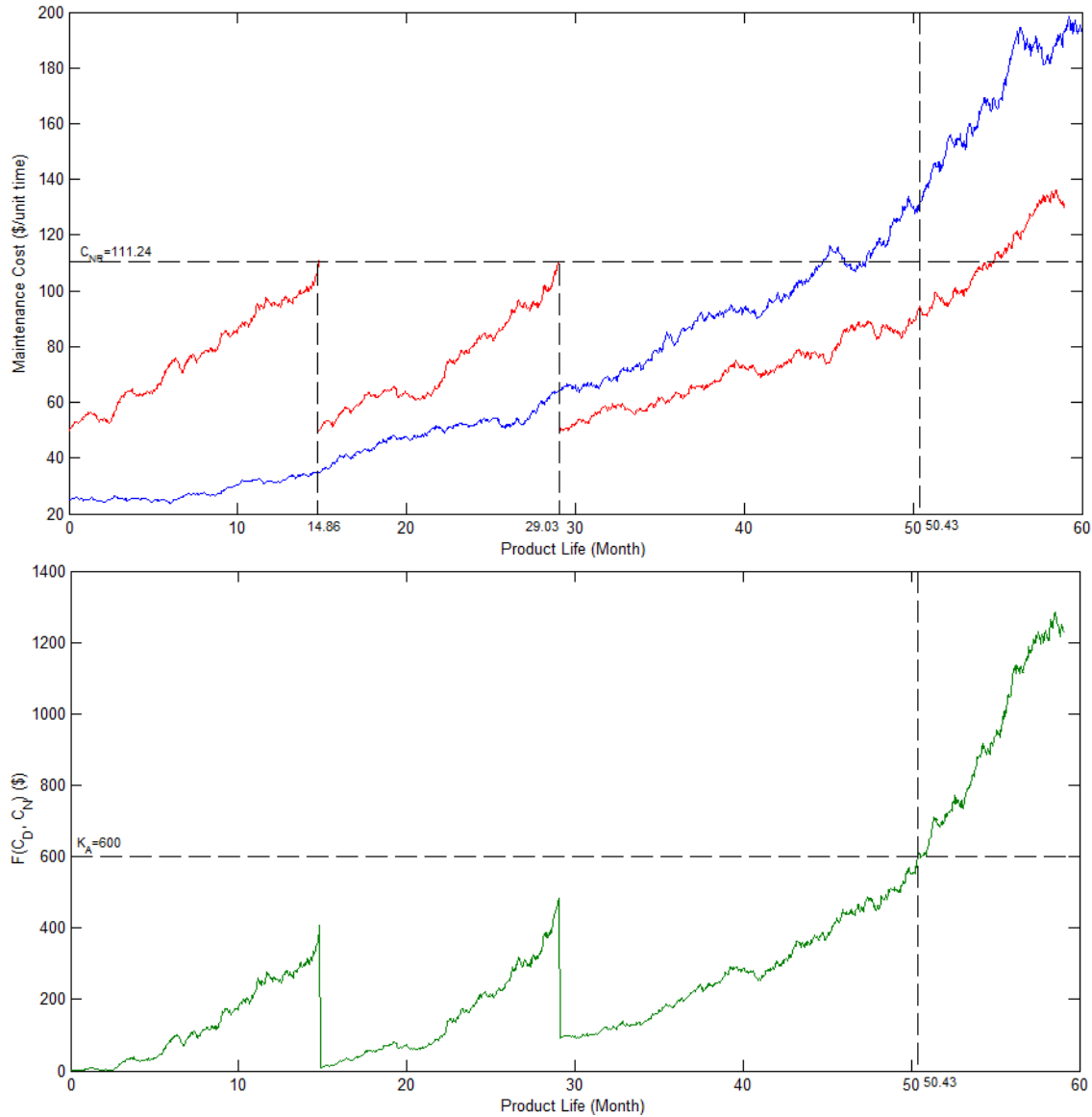
Figure 12. Sample Paths of O&M Costs and F(C_D, C_N) in 60 Months

To better characterize the product remanufacturing and replacement decisions numerically, we use Monte Carlo method to simulate the GBM processes of both O&M costs. Figure 12 displays the sample paths of the durable-part O&M cost (i.e., blue line), the non-durable-part O&M cost (i.e., red line), and the derived path of project-value-difference function $F(C_D, C_N)$ (i.e., green line). In this sample case, the remanufacturing option threshold (i.e., C_N =111.24) is first reached around the 15th month, and hence, the product is remanufactured for the first time.

Sequentially, the remanufacturing option threshold is reached for the second time around the 29th month. We note that, till the 29th month, the project-value-difference function is below the manufacturing cost K_A=$600, and hence, the replacement option is yet exercised. Around the 50th month, the difference between the new project value and the current project value hits the manufacturing cost threshold, and hence, the product is retired and replaced with a new one. The virtual paths after the replacement point (the 50.43th month) are given here to provide a complete picture about how the O&M costs and the derived project-value-difference function evolve with respect to time. By the virtual path, it can be observed that, the point when the remanufacturing option threshold is reached for the third time (around the 54th month) comes after the replacement. Hence, the product is remanufactured twice before being retired and replaced. Furthermore, we note that, when the replacement option is exercised, the durable-part O&M cost is approximately $130 per month, which is greater than the lower bound of $67.46 per month given in Table 2.

Government Sustainable Policy

From a social planner's perspective, remanufacturing is one of the most important sustainable processes to reduce raw materials consumption and conserve energy as well as landfill space. Therefore, the usage of subsidies as an economic instrument to incentivize and facilitate the remanufacturing processes has significantly increased at all of local and national levels. For instance, in California, remanufacturers are subsidized $0.28 per pound for remanufacturing the electronic products. In New York, remanufacturers receive tax credits that are commensurate with the number of employees and/or the durability of capital investment. However, critical questions such as whether the remanufacturing subsidies are environmentally viable have not

been well answered by either governmental regulators or academics. Hence, in this study, we numerically examine the impact of the remanufacturing subsidy on the amount of wastes (in weight) disposed at landfill.

Assumption 5.4   *Through remanufacturing, α fraction of the product (in weight) is re-placed and disposed at landfill with no cost, and the weight of a unit product is W (lb).*

Given this assumption, if a product was remanufactured N times before being replaced, the total landfill disposal throughout the product life-cycle is $(N\alpha+1)W$. The simplifying assumption of a negligible landfill disposal cost enables us to focus on the remanufacturing and the re-placement decisions without being distracted by how the disposal cost influences these decisions, which is beyond the scope of this study. Moreover, mathematically, the disposal cost can be easily incorporated into the remanufacturing cost without affecting any analytical results presented so far.

Assumption 5.5   *The government is subsidizing the firm S ($/unit product) for remanufac-turing a product.*

By Assumption 5.5, from the firm's perspective, receiving a remanufacturing subsidy is equivalent to a decrease in the remanufacturing cost. Furthermore, the impact of a higher re-manufacturing subsidy on the firm's remanufacturing and replacement decisions is two-fold:

(i) A shorter life of the non-durable parts – according to Corollary 5.1, a lower remanufacturing cost (i.e., a higher remanufacturing subsidy) results in a lower threshold of non-durable-part O&M cost for the remanufacturing option. Hence, the product is remanufactured more fre-quently, and the expected life of the non-durable parts is shortened.

(ii) A longer life of the durable parts – accordingly to Corollary 5.3, a lower remanufacturing cost (i.e., a higher remanufacturing subsidy) results in a higher lower bound of the replacement option threshold. Hence, the product is retired and replaced less frequently, and the expected life of the durable parts is prolonged.

Given these two sustainably conflicting impacts, it is desirable to investigate the total environmental impact resulted from a remanufacturing subsidy in terms of the amount of wastes disposed at landfills. Toward this goal, we consider three scenarios: (i) without remanufacturing subsidy, (ii) remanufacturing subsidy S = \$20, and (iii) remanufacturing subsidy S = \$40. Therefore, while all the other parameter values remain the same as given in Table 1, the resulted remanufacturing costs become K_R = \$250, K_R = \$230, and K_R = \$210, respectively. Furthermore, the GBM paths were generated 1,000 times for each scenario, and the disposal fraction in the remanufacturing process $\alpha=0.5$ (see Assumption 5.4). The experimental results are summarized in Table 3, and the symbols used in Table 3 are defined after the table.

Table 5. Experimental Results of Monte Carlo Simulation

$M_N (T_N)$	K_R = \$250 (S=\$0)	K_R = \$230 (S=\$20)	K_R = \$210 (S=\$40)
$M_0 (T_0)$	3 (18.23)	1 (16.17)	0 (0.00)
$M_1 (T_1)$	125 (31.76)	59 (31.93)	14 (30.01)
$M_2 (T_2)$	677 (46.73)	538 (45.22)	368 (43.31)
$M_3 (T_3)$	194 (54.14)	395 (54.04)	597 (52.92)
$M_4 (T_4)$	1 (66.16)	7 (65.89)	21 (64.61)
T	46.23	48.04	49.31
L	2.03W	2.17W	2.31W
LPUT	0.0451	0.0437	0.0479

M_N the number of scenarios where the product is remanufactured N times before being replaced.

T_N the average product life (month) of the scenarios where the product is remanufactured N times   before being replaced.

T the average product life (month) of all the scenarios, $T = \frac{1}{1000}\sum_N M_N T_N$

L the average amount of landfill disposal (lb) of all the scenarios, $L = \frac{1}{1000}\sum_N M_N (N\alpha + 1)W$

$LPUT$ the average landfill disposal per unit time (lb/month) of all the scenarios,

$LPUT = \frac{1}{1000}\sum_{k=1}^{1000}\frac{(N_k\alpha + 1)W}{T_k}$ where N_k and T_k represent the number of remanufacturing and

the product life in scenarios k, respectively.

The following observations can be derived from the numerical results presented in Table 3.

Observation 1: in our experiment, the number of remanufacturing throughout a product life-cycle tends to be higher in the scenario with a lower remanufacturing cost (i.e., a higher remanufacturing subsidy).

Observation 2: in our experiment, a lower remanufacturing cost (i.e., a higher remanufacturing subsidy) results in a higher average product life. Intuitively, with a lower remanufacturing cost, the product is remanufactured more frequently. Therefore, the non-durable-part O&M cost is kept in a relatively low level, which incentivizes the firm to operate the product for longer.

Observation 3: in our experiment, a lower remanufacturing cost (i.e., a higher remanufacturing subsidy) results in a higher level of the landfill disposal throughout the product life-cycle. Intuitively, with a lower remanufacturing cost, there is a higher chance for the product to go

through more remanufacturing processes before being replaced. Therefore, more non-durable parts may be consumed and disposed throughout the product life-cycle.

Observation 4: in our experiment, the lowest average landfill disposal per unit time (LPUT) is observed when the remanufacturing subsidy is set to $20 (i.e., remanufacturing cost is $230). This implies that introducing a new remanufacturing subsidy or increasing the existing remanufacturing subsidy may not always result in less landfill disposal per unit time. Hence, any economic instrument of a similar nature as the remanufacturing subsidy described in this experiment needs to be carefully examined by the policy makers before its implemention.

By running additional simulation experiments, more numerical results can be generated and analyzed to gain further observations. Also, statistical methods can be utilized to obtain more general conclusions regarding how the remanufacturing subsidy influences the firm's remanufacturing and replacement decisions, as well as the corresponding landfill disposal.

5.4   Managerial Insights and Policy Implications

In this chapter, we considered the uncertainties in O&M costs for both durable parts and non-durable parts of a product. By modeling these O&M costs as GBM processes, we investigated a firm's product remanufacturing and replacement decisions from a real options perspective. We also examined the impact of government's remanufacturing subsidy on these decisions and the corresponding landfill disposal. Via analytical and numerical studies, a series of managerial insights and policy implication were derived. Specifically, as the O&M costs become more volatile, it is beneficial to defer the exercise of the remanufacturing option and the replacement option because the option values are higher. Also, with a higher remanufacturing cost, it is expected that the product will be replaced earlier, which implies a shorter product life.

Furthermore, from a government policy maker's perspective, increasing the remanufacturing subsidy incentivizes the firm to remanufacture the product more frequently, which implies a shorter life of the non-durable parts (which are replaced in remanufacturing process). On the other hand, given the more frequent remanufacturing, the overall O&M cost of the product is kept in a relatively low level, and it is beneficial for the firm to operate the product for longer, which implies a longer life of the durable parts. Given these conflicting ramifications, it is advised that the government not unconditionally advocate the remanufacturing process via economic incentives. As numerically demonstrated in this study, in certain cases, a higher remanufacturing subsidy may result in more landfill disposal per unit time, which is probably against the governmental policy maker's original intention.

CHAPTER 6

DISCUSSIONS

First of all, we note that the statement "See Appendix A for proof" in the paragraph after Assumption 3.9 can be considered as imprecise. The precise statement should be "See Appendix A for the deduction of these conditions". Also, we note that the convergence condition $\rho > \mu$ in Chapter 4 is necessary for analytical investigations, which can be widely observed in literature [59][84][85][86].

Furthermore, in Chapter 4, the maintenance cost is assumed to proportional to the remanufacturing cost. This is because the primary goal of maintenance is to preserve the reliable operation of the product that is providing services. The maintenance process is subject to costs associated with inspection, adjustment, and component-replacement. Typically, the maintenance cost increases with operating time as the components gradually deteriorate [55][56]. Since the maintenance cost and the remanufacturing cost share similar costing factors such as cleaning, testing as well as component-replacement, and both increases with the operating time, the maintenance cost is assumed to be linear to the remanufacturing cost C [57][58]. Also, the possible relaxation of this assumption can be considered as one of the future works.

CHAPTER 7

CONCLUSIONS

In this article, three research works regarding the critical decisions in sustainable processes such as product recycling and remanufacturing were presented. Based on these research works, a series of managerial insights and policy implications were derived from analytical and numerical analyses. Specifically,

(1) In the first study, we formulated and analyzed CLSC models for a manufacturer and a collector with variable product weight and collection rate. We demonstrated that there is an inherent conflict between two environmental factors of the product weight and the collection rate, which implies that manufacturers in practice may not be able to pursue both a lower level of the product weight and a higher level of the collection rate simultaneously, and efforts to improve one environmental factor may lead to an unintended negative impact on the other factor. Consequently, under certain conditions (e.g., the marginal recycling benefit or collecting subsidy is sufficiently high), the landfill quantity increases with the marginal recycling benefit or collecting subsidy. Moreover, the landfill quantity in the centralized CLSC can be achieved or even reduced in the decentralized CLSC via appropriate financial instruments. All in all, implementing the subsidy/fee system in CLSC's may not always be environmentally desirable, and may result in more landfill quantity relative to the non-subsidy/fee scenario, especially when the subsidy is sufficiently high.

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(2) In the second study, from a real options perspective, we modeled and analyzed the remanufacturing decision making by a manufacturer who leases a product to a service provider when the remanufacturing cost follows a GBM process. By deriving the appropriate optimal timing of the remanufacturing and performing sensitivity analysis with and without the government subsidy and fee, a series of managerial insights and policy implications were derived, and numerically illustrated. Some of the critical findings are: (i) under certain conditions, increasing the remanufacturing subsidy or the disposal fee will result in more raw materials consumption and landfill disposal, which is against the intention of the policy makers introducing these economic instruments; (ii) if the remanufacturing cost becomes more volatile, it is advised that the policy maker should increase the remanufacturing subsidy or reduce the disposal fee to avoid more negative environmental impacts.

(3) In the third study, we considered the uncertainties in O&M costs for both durable parts and non-durable parts of a product. By modeling these O&M costs as GBM processes, we investigated a firm's product remanufacturing and replacement decisions from a real options perspective. We also examined the impact of government's remanufacturing subsidy on these decisions and the corresponding landfill disposal. Via analytical and numerical studies, a series of managerial insights and policy implication were derived. Specifically, as the O&M costs become more volatile, it is beneficial to defer the exercise of the remanufacturing option and the replacement option because the option values are higher. Also, with a higher remanufacturing cost, it is expected that the product will be replaced earlier, which implies a shorter product life.

Given the less than stable current economic condition and the increasing sustainable efforts from industries, government, and environmental groups, it is highly desirable to understand the critical decisions in terms of sustainable product design, used-product collection, and remanufacturing relevant investments. Toward this goal, the questions raised and addressed in this article is timely (if not urgent), and we hope that it can serve as a reference point to stimulate the discussion on this important topic of government policies toward remanufacturing in particular and sustainability in general.

Moreover, this paper provides the basis for a number of interesting extensions. For instance, a menu of options such as refurbishing and demanufacturing can be concurrently considered along with recycling and remanufacturing. Other interesting extensions of our models include (i) consider the competition among multiple manufacturers and collectors in closed loop supply chains, (ii) utilizing backward optimization approach to investigate the scenario where the remanufacturing and replacement option expires after some time point (e.g., a hard-stop of product life is defined in the leasing contract); (iii) formulating a Stackelberg game to consider the competition between manufacturers and service providers.

APPENDIX

A. Deduction of Conditions in Assumption 3.9

In the CR model, if $\beta > \sqrt{8\gamma S}$ and $\Delta > Ka_0/S$, we have

$$p_{CR} = \frac{3\beta - \sqrt{\beta^2 - 8\gamma S}}{4\gamma} > \frac{2\beta}{4\gamma} > 0, \quad w_{CR} = \frac{(S-K)(\beta - \sqrt{\beta^2 - 8\gamma S})}{2\gamma S(a_0 - \Delta)} > \frac{(S-K)(\beta - \beta)}{2\gamma S(a_0 - \Delta)} = 0$$

$$\tau_{CR} = 1 - \frac{K(a_0 - \Delta)}{\Delta(S-K)} > 1 - \frac{K(a_0 - \Delta)}{\Delta(Ka_0/\Delta - K)} = 0$$

Also, according to Assumption 3.6 and 3.10, obviously, $\tau_{CR} < 1$. Similarly, in the DR model, if $\beta > \sqrt{8\gamma S}$

and $\Delta > Ka_0/S$, we have

$$p_{DR} = \frac{3\beta - \sqrt{\beta^2 - 8\gamma S}}{4\gamma} > 0, \quad w_{DR} = \frac{2S(\beta - \sqrt{\beta^2 - 8\gamma S})}{\gamma[\sqrt{K\Delta} + \sqrt{K\Delta + 4S(a_0 - \Delta)}]^2} > 0,$$

$$\tau_{DR} = 1 - \frac{K}{2S} - \frac{\sqrt{K^2\Delta^2 + 4SK\Delta(a_0 - \Delta)}}{2S\Delta} > 1 - \frac{K}{2\frac{Ka_0}{\Delta}} - \frac{\sqrt{K^2\Delta^2 + 4\frac{Ka_0}{\Delta}K\Delta(a_0 - \Delta)}}{2\frac{Ka_0}{\Delta}\Delta} = 1 - \frac{\Delta}{2a_0} - \frac{2a_0 - \Delta}{2a_0} = 0$$
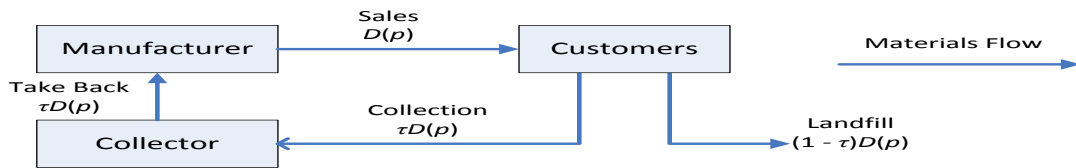
Also, obviously, $\tau_{DR} > 0$. Similarly, in the DR-G model, if $\beta > \sqrt{8\gamma S}$ and $\Delta + \sigma > \frac{K(a_0 + \alpha)}{S}$, we have

$$p_{DR\text{-}G} = \frac{3\beta - \sqrt{\beta^2 - 8\gamma S}}{4\gamma} > 0, \quad w_{DR\text{-}G} = \frac{2S(\beta - \sqrt{\beta^2 - 8\gamma S})}{\gamma[\sqrt{K(\Delta + \sigma)} + \sqrt{K(\Delta + \sigma) + 4S(a_0 - \Delta + \alpha - \sigma)}]^2}$$

$$\tau_{DR\text{-}G} = 1 - \frac{K}{2S} - \frac{\sqrt{K^2(\Delta + \sigma)^2 + 4SK(\Delta + \sigma)(a_0 - \Delta + \alpha - \sigma)}}{2S(\Delta + \sigma)}$$

$$> 1 - \frac{K}{2\frac{K(a_0 + \alpha)}{\Delta + \sigma}} - \frac{\sqrt{K^2(\Delta + \sigma)^2 + 4\frac{K(a_0 + \alpha)}{\Delta + \sigma}K(\Delta + \sigma)(a_0 - \Delta + \alpha - \sigma)}}{2\frac{K(a_0 + \alpha)}{\Delta + \sigma}(\Delta + \sigma)} = 1 - \frac{\Delta + \sigma}{2(a_0 + \alpha)} - \frac{2(a_0 + \alpha) - (\Delta + \sigma)}{2(a_0 + \alpha)} = 0$$

B. Proof for Proposition 3.1

Let $\Pi_{CR} - \Pi_{NR} = H(\Delta) = S\ln\frac{a_0(S-K)}{S(a_0 - \Delta)} - K\ln\frac{\Delta(S-K)}{K(a_0 - \Delta)}$, we have $\frac{dH}{d\Delta} = \frac{S\Delta - Ka_0}{\Delta(a_0 - \Delta)} > 0$ (Assumption 3.6 & 3.9).

Hence, according to Assumption 3.9, $\Pi_{CR} - \Pi_{NR} = H(\Delta) > H(\frac{Ka_0}{\Delta}) = \frac{Ka_0}{\Delta}\ln\frac{a_0(Ka_0/\Delta - K)}{Ka_0(a_0 - \Delta)/\Delta} - K\ln\frac{\Delta(Ka_0/\Delta - K)}{K(a_0 - \Delta)} = 0$.

$$\Pi_{DR}^M - \Pi_{NR} = \frac{(\beta + \sqrt{\beta^2 - 8\gamma S})^2}{16\gamma} + K - \frac{2SK\Delta}{K\Delta + \sqrt{K^2\Delta^2 + 4SK\Delta(a_0 - \Delta)}} + S\ln\frac{2S(\beta - \sqrt{\beta^2 - 8\gamma S})}{\gamma w_0[\sqrt{K\Delta} + \sqrt{K\Delta + 4S(a_0 - \Delta)}]^2}$$

$$- \left[\frac{(\beta + \sqrt{\beta^2 - 8\gamma S})^2}{16\gamma} + S\ln\frac{\beta - \sqrt{\beta^2 - 8\gamma S}}{2\gamma a_0 w_0}\right] = K - \frac{2SK\Delta}{K\Delta + \sqrt{K^2\Delta^2 + 4SK\Delta(a_0 - \Delta)}} + S\ln\frac{4Sa_0}{[\sqrt{K\Delta} + \sqrt{K\Delta + 4S(a_0 - \Delta)}]^2}$$

Let $\Pi_{DR}^M - \Pi_{NR} = F(\Delta) = K - \frac{2SK\Delta}{K\Delta + \sqrt{K^2\Delta^2 + 4SK\Delta(a_0 - \Delta)}} + S\ln\frac{4Sa_0}{[\sqrt{K\Delta} + \sqrt{K\Delta + 4S(a_0 - \Delta)}]^2}$, we have

$$\frac{dH}{d\Delta} = 2S\left[-\frac{K\sqrt{K\Delta}[\sqrt{K\Delta} + \sqrt{K\Delta + 4S(a_0 - \Delta)}] - K\Delta\left[K + \frac{2K^2\Delta + 4SK(a_0 - 2\Delta)}{2\sqrt{K\Delta}\sqrt{K\Delta + 4S(a_0 - \Delta)}}\right]}{K\Delta[\sqrt{K\Delta} + \sqrt{K\Delta + 4S(a_0 - \Delta)}]^2} - \frac{\frac{\sqrt{K}}{2\sqrt{\Delta}} + \frac{K - 4S}{2\sqrt{K\Delta + 4S(a_0 - \Delta)}}}{[\sqrt{K\Delta} + \sqrt{K\Delta + 4S(a_0 - \Delta)}]}\right]$$

$$= \left[\frac{2S}{[\sqrt{K\Delta} + \sqrt{K\Delta + 4S(a_0 - \Delta)}]}\right]\left[-\frac{K\Delta[K\Delta + 4S(a_0 - \Delta)] - K^2\Delta^2 - 2SK\Delta(a_0 - 2\Delta)}{\Delta[\sqrt{K\Delta} + \sqrt{K\Delta + 4S(a_0 - \Delta)}]K\Delta\sqrt{K\Delta + 4S(a_0 - \Delta)}} - \frac{\sqrt{K}\sqrt{K\Delta + 4S(a_0 - \Delta)} + (K - 4S)\sqrt{\Delta}}{2\sqrt{\Delta}\sqrt{K\Delta + 4S(a_0 - \Delta)}}\right]$$

$$= \frac{2S\left[-4S\sqrt{K}a_0 + [(4S - K)\sqrt{\Delta} - \sqrt{K}\sqrt{K\Delta + 4S(a_0 - \Delta)}][\sqrt{K\Delta} + \sqrt{K\Delta + 4S(a_0 - \Delta)}]\right]}{2[\sqrt{K\Delta} + \sqrt{K\Delta + 4S(a_0 - \Delta)}]^2\sqrt{\Delta}\sqrt{K\Delta + 4S(a_0 - \Delta)}}$$

$$= \frac{2S[(2S - K)\sqrt{\Delta} - \sqrt{K}\sqrt{K\Delta + 4S(a_0 - \Delta)}]}{\sqrt{\Delta}[\sqrt{K\Delta} + \sqrt{K\Delta + 4S(a_0 - \Delta)}]^2[K\Delta + 4S(a_0 - \Delta)]}$$

Hence, $\frac{dH}{d\Delta} > 0 \Leftrightarrow (2S - K)\sqrt{\Delta} - \sqrt{K}\sqrt{K\Delta + 4S(a_0 - \Delta)} > 0 \Leftrightarrow (4S^2 - 4SK + K^2)\Delta > K[K\Delta + 4S(a_0 - \Delta)] > 0 \Leftrightarrow \Delta > \frac{Ka_0}{S}$,

which is guaranteed by Assumption 3.9. Hence, we have

$$\Pi_{DR}^M - \Pi_{NR} = H(\Delta) > H(\frac{Ka_0}{S}) = K - \frac{2SK\frac{Ka_0}{S}}{K\frac{Ka_0}{S} + \sqrt{K^2(\frac{Ka_0}{S})^2 + 4SK\frac{Ka_0}{S}(a_0 - \frac{Ka_0}{S})}} + S\ln\frac{4Sa_0}{[\sqrt{K\frac{Ka_0}{S}} + \sqrt{K\frac{Ka_0}{S} + 4S(a_0 - \frac{Ka_0}{S})}]^2}$$

$$= K - \frac{2SK}{K + 2S - K} + S\ln\frac{4S^2}{[K + 2S - K]^2} = 0$$

C. Proof for Proposition 3.2

$$\frac{\partial w_{DR}}{\partial \Delta} = \frac{2S(\beta - \sqrt{\beta^2 - 8\gamma S})[(4S - K)\sqrt{\Delta} - \sqrt{K}\sqrt{K\Delta + 4S(a_0 - \Delta)}]}{\gamma\sqrt{K\Delta^2 + 4S\Delta(a_0 - \Delta)}[\sqrt{K\Delta} + \sqrt{K\Delta + 4S(a_0 - \Delta)}]^3} > 0 \quad \Leftrightarrow \quad (4S - K)\sqrt{\Delta} - \sqrt{K}\sqrt{K\Delta + 4S(a_0 - \Delta)} > 0$$

$$\Leftrightarrow \quad (4S - K)\sqrt{\Delta} > \sqrt{K}\sqrt{K\Delta + 4S(a_0 - \Delta)} \quad \Leftrightarrow \quad \Delta > \frac{Ka_0}{S}$$

which is guaranteed by Assumption 3.9.

$$\frac{\partial \tau_{DR}}{\partial \Delta} = -\frac{\dfrac{\Delta[2K^2\Delta + 4SK(a_0 - 2\Delta)]}{2\sqrt{K^2\Delta^2 + 4SK\Delta(a_0 - \Delta)}} - \sqrt{K^2\Delta^2 + 4SK\Delta(a_0 - \Delta)}}{2S\Delta^2} = \frac{Ka_0}{\Delta\sqrt{K^2\Delta^2 + 4SK\Delta(a_0 - \Delta)}} > 0$$

D. Proof for Proposition 3.3

Since $\dfrac{\partial L_{DR}}{\partial \Delta} = -\dfrac{2K^2 S\left[K\Delta + 2S(a_0 - 2\Delta) + \sqrt{K^2\Delta^2 + 4SK\Delta(a_0 - \Delta)}\right]}{[K\Delta + \sqrt{K^2\Delta^2 + 4SK\Delta(a_0 - \Delta)}]^2\sqrt{K^2\Delta^2 + 4SK\Delta(a_0 - \Delta)}}$, we have

$$\frac{\partial L_{DR}}{\partial \Delta} < 0 \Leftrightarrow K\Delta + 2S(a_0 - 2\Delta) + \sqrt{K^2\Delta^2 + 4SK\Delta(a_0 - \Delta)} > 0$$

If $\Delta \le a_0/2$, then obviously, $K\Delta + 2S(a_0 - 2\Delta) + \sqrt{K^2\Delta^2 + 4SK\Delta(a_0 - \Delta)} > 0$, and $\dfrac{\partial L_{DR}}{\partial \Delta} < 0$ accordingly.

If $\Delta > a_0/2$, $\dfrac{\partial L_{DR}}{\partial \Delta} < 0 \Leftrightarrow \sqrt{K^2\Delta^2 + 4SK\Delta(a_0 - \Delta)} > 2S(2\Delta - a_0) - K\Delta \Leftrightarrow K\Delta^2 > S(2\Delta - a_0)^2 \Leftrightarrow \Delta < \dfrac{\sqrt{S}a_0}{2\sqrt{S} - \sqrt{K}}$

Obviously, $\dfrac{\sqrt{S}a_0}{2\sqrt{S} - \sqrt{K}} > \dfrac{a_0}{2}$ and $\dfrac{\sqrt{S}a_0}{2\sqrt{S} - \sqrt{K}} < \dfrac{\sqrt{S}a_0}{2\sqrt{S} - \sqrt{S}} = a_0$. Hence, if $\dfrac{a_0}{2} < \Delta < \dfrac{\sqrt{S}a_0}{2\sqrt{S} - \sqrt{K}}$, $\dfrac{\partial L_{DR}}{\partial \Delta} < 0$; if

$\dfrac{\sqrt{S}a_0}{2\sqrt{S} - \sqrt{K}} < \Delta < a_0$, $\dfrac{\partial L_{DR}}{\partial \Delta} > 0$. Combine two cases, $\dfrac{\partial L_{DR}}{\partial \Delta} < 0$ if $\Delta < \dfrac{\sqrt{S}a_0}{2\sqrt{S} - \sqrt{K}}$; $\dfrac{\partial L_{DR}}{\partial \Delta} > 0$ if $\dfrac{\sqrt{S}a_0}{2\sqrt{S} - \sqrt{K}} < \Delta < a_0$

E. Proof for Proposition 3.4

$$w_{NR} - w_{DR} = \frac{\beta - \sqrt{\beta^2 - 8\gamma S}}{2\gamma a_0} - \frac{2S(\beta - \sqrt{\beta^2 - 8\gamma S})}{\gamma[\sqrt{K\Delta} + \sqrt{K\Delta + 4S(a_0 - \Delta)}]^2} = \frac{(\beta - \sqrt{\beta^2 - 8\gamma S})\left[K\Delta + \sqrt{K\Delta}\sqrt{K\Delta + 4S(a_0 - \Delta)} - 2S\Delta\right]}{\gamma a_0[\sqrt{K\Delta} + \sqrt{K\Delta + 4S(a_0 - \Delta)}]^2} < 0$$

$$\Leftrightarrow K\Delta + \sqrt{K\Delta}\sqrt{K\Delta + 4S(a_0 - \Delta)} - 2S\Delta < 0 \Leftrightarrow Ka_0 < S\Delta$$

$$w_{DR} - w_{CR} = -\frac{\left[\beta - \sqrt{\beta^2 - 8\gamma S}\right]\left[(S - K)\sqrt{K\Delta}\sqrt{K\Delta + 4S(a_0 - \Delta)} - K^2\Delta - SK(2a_0 - 3\Delta)\right]}{\gamma S(a_0 - \Delta)[\sqrt{K\Delta} + \sqrt{K\Delta + 4S(a_0 - \Delta)}]^2} < 0$$

$$\Leftrightarrow (S - K)\sqrt{K\Delta}\sqrt{K\Delta + 4S(a_0 - \Delta)} > K^2\Delta + SK(2a_0 - 3\Delta) \Leftrightarrow S\Delta > Ka_0$$

$$\tau_{DR} - \tau_{CR} = \frac{K(a_0 - \Delta)}{\Delta(S - K)} - \frac{K}{2S} - \frac{\sqrt{K^2\Delta^2 + 4KS\Delta(a_0 - \Delta)}}{2S\Delta} < 0 \Leftrightarrow S\Delta > Ka_0$$

$$L_{NR} - L_{DR} = \frac{S}{a_0} - \frac{2KS}{K\Delta + \sqrt{K^2\Delta^2 + 4SK\Delta(a_0 - \Delta)}} > 0 \Leftrightarrow S\Delta > Ka_0$$

$$L_{DR} - L_{CR} = \frac{2KS}{K\Delta + \sqrt{K^2\Delta^2 + 4SK\Delta(a_0 - \Delta)}} - \frac{K}{\Delta} > 0 \Leftrightarrow S\Delta > Ka_0$$

These conditions are all guaranteed by Assumption 3.9.

F. Proof for Proposition 3.5

$$\frac{\partial w_{DR\text{-}G}}{\partial \sigma} = \frac{\frac{4S-K}{\sqrt{K(\Delta+\sigma)+4S(a_0-\Delta+\alpha-\sigma)}} - \frac{K}{\sqrt{K(\Delta+\sigma)}}}{[\sqrt{K(\Delta+\sigma)}+\sqrt{K(\Delta+\sigma)+4S(a_0-\Delta+\alpha-\sigma)}]^3} = \frac{2S(\beta-\sqrt{\beta^2-8\gamma S})\left[\frac{4S-K}{\sqrt{K(\Delta+\sigma)+4S(a_0-\Delta+\alpha-\sigma)}} - \frac{\sqrt{K}}{\sqrt{\Delta+\sigma}}\right]}{\gamma[\sqrt{K(\Delta+\sigma)}+\sqrt{K(\Delta+\sigma)+4S(a_0-\Delta+\alpha-\sigma)}]^3} > 0$$

$$\Leftrightarrow \quad \frac{4S-K}{\sqrt{K(\Delta+\sigma)+4S(a_0-\Delta+\alpha-\sigma)}} - \frac{\sqrt{K}}{\sqrt{\Delta+\sigma}} > 0 \quad \Leftrightarrow \quad 2S(\Delta+\sigma)+S(\Delta+\sigma)-K(a_0+\alpha)+(S-K)(\Delta+\sigma) > 0$$

which is guaranteed by Assumption 3.9.

$$\frac{\partial \tau_{DR\text{-}G}}{\partial \sigma} = \frac{K(a_0+\alpha)}{(\Delta+\sigma)\sqrt{K^2(\Delta+\sigma)^2+4SK(\Delta+\sigma)(a_0-\Delta+\alpha-\sigma)}} > 0$$

G. Proof for Proposition 3.6

Since $\dfrac{\partial L_{DR\text{-}G}}{\partial \sigma} = -\dfrac{2K^2S\left[K(\Delta+\sigma)+2S[a_0+\alpha-2(\Delta+\sigma)]+\sqrt{K^2(\Delta+\sigma)^2+4SK(\Delta+\sigma)(a_0-\Delta+\alpha-\sigma)}\right]}{[K(\Delta+\sigma)+\sqrt{K^2(\Delta+\sigma)^2+4SK(\Delta+\sigma)(a_0-\Delta+\alpha-\sigma)}]^2\sqrt{K^2(\Delta+\sigma)^2+4SK(\Delta+\sigma)(a_0-\Delta+\alpha-\sigma)}}$, we

have $\dfrac{\partial L_{DR\text{-}G}}{\partial \sigma} < 0 \Leftrightarrow K(\Delta+\sigma)+2S[a_0+\alpha-2(\Delta+\sigma)]+\sqrt{K^2(\Delta+\sigma)^2+4SK(\Delta+\sigma)(a_0-\Delta+\alpha-\sigma)} > 0$.

If $\sigma \le \dfrac{a_0+\alpha}{2}-\Delta$, then obviously, $\dfrac{\partial L_{DR}}{\partial \Delta} < 0$.

If $\sigma > \dfrac{a_0+\alpha}{2}-\Delta$, $\dfrac{\partial L_{DR\text{-}G}}{\partial \sigma} < 0 \Leftrightarrow \sqrt{K^2(\Delta+\sigma)^2+4SK(\Delta+\sigma)(a_0-\Delta+\alpha-\sigma)} > 2S[2(\Delta+\sigma)-a_0+\alpha]-K(\Delta+\sigma)$

$$\Leftrightarrow \quad K^2(\Delta+\sigma)^2+4SK(\Delta+\sigma)(a_0-\Delta+\alpha-\sigma) > 4S^2[2(\Delta+\sigma)-a_0+\alpha]^2-4SK(\Delta+\sigma)[2(\Delta+\sigma)-a_0+\alpha]+K^2(\Delta+\sigma)^2$$

$$\Leftrightarrow \quad K(\Delta+\sigma)^2 > S[2(\Delta+\sigma)-a_0+\alpha]^2 \quad \Leftrightarrow \quad \sqrt{K}(\Delta+\sigma) > \sqrt{S}[2(\Delta+\sigma)-a_0+\alpha] \quad \Leftrightarrow \quad \sigma < \frac{\sqrt{S}(a_0+\alpha)}{2\sqrt{S}-\sqrt{K}}-\Delta$$

Obviously, $\dfrac{\sqrt{S}(a_0+\alpha)}{2\sqrt{S}-\sqrt{K}}-\Delta > \dfrac{\sqrt{S}(a_0+\alpha)}{2\sqrt{S}}-\Delta = \dfrac{a_0+\alpha}{2}-\Delta$. Hence, if $\dfrac{a_0+\alpha}{2}-\Delta < \sigma < \dfrac{\sqrt{S}(a_0+\alpha)}{2\sqrt{S}-\sqrt{K}}-\Delta$, $\dfrac{\partial L_{DR\text{-}G}}{\partial \sigma} < 0$; if

$\sigma > \dfrac{\sqrt{S}(a_0+\alpha)}{2\sqrt{S}-\sqrt{K}}-\Delta$, $\dfrac{\partial L_{DR-G}}{\partial \sigma} > 0$.

Combine the cases, we have $\dfrac{\partial L_{DR-G}}{\partial \sigma} < 0$ if $\sigma < \dfrac{\sqrt{S}(a_0+\alpha)}{2\sqrt{S}-\sqrt{K}}-\Delta$, and $\dfrac{\partial L_{DR-G}}{\partial \sigma} > 0$ if $\sigma > \dfrac{\sqrt{S}(a_0+\alpha)}{2\sqrt{S}-\sqrt{K}}-\Delta$.

H. Proof for Propositions 3.7

$$w_{DR} - w_{DR\text{-}G} = \frac{2S(\beta - \sqrt{\beta^2 - 8\gamma S})}{\gamma[\sqrt{K\Delta} + \sqrt{K\Delta + 4S(a_0 - \Delta)}]^2} - \frac{2S(\beta - \sqrt{\beta^2 - 8\gamma S})}{\gamma[\sqrt{K(\Delta + \sigma)} + \sqrt{K(\Delta + \sigma) + 4S(a_0 - \Delta + \alpha - \sigma)}]^2} < 0$$

$$\Leftrightarrow \Delta\sqrt{[K(\Delta + \sigma) - 2S\sigma]^2 + 4S[K(\Delta + \sigma)(a_0 - \Delta + \alpha) - S\sigma^2]} < (\Delta + \sigma)\sqrt{K^2\Delta^2 + 4KS\Delta(a_0 - \Delta)} \quad \Leftrightarrow \quad a_0\sigma > \Delta\alpha$$

$$\tau_{DR} - \tau_{DR\text{-}G} = \frac{\sqrt{K^2(\Delta + \sigma)^2 + 4SK(\Delta + \sigma)(a_0 - \Delta + \alpha - \sigma)}}{2S(\Delta + \sigma)} - \frac{\sqrt{K^2\Delta^2 + 4KS\Delta(a_0 - \Delta)}}{2S\Delta} < 0 \Leftrightarrow a_0\sigma > \Delta\alpha$$

Since $a_0 > \Delta$ (Assumption 3.9) and $\sigma = \frac{\alpha}{\tau} > \alpha$ (Assumption 3.10), $a_0\sigma > \Delta\alpha$.

I. Proof for Propositions 3.8

$$L_{DR\text{-}G} < L_{CR} \Leftrightarrow \frac{2KS}{K(\Delta + \sigma) + \sqrt{K^2(\Delta + \sigma)^2 + 4SK(\Delta + \sigma)(a_0 - \Delta + \alpha - \sigma)}} - \frac{K}{\Delta}$$

$$\Leftrightarrow 2S\Delta - K(\Delta + \sigma) < \sqrt{K^2(\Delta + \sigma)^2 + 4SK(\Delta + \sigma)(a_0 - \Delta + \alpha - \sigma)} \Leftrightarrow S\Delta^2 < K(\Delta + \sigma)(a_0 + \alpha - \sigma)$$

Under the revenue-neutrality requirement, we have

$$\alpha\tau_{DR\text{-}G} = \sigma \Leftrightarrow \alpha = \frac{\sigma[2S(\Delta + \sigma) - K\Delta] - \sigma\sqrt{[2S(\Delta + \sigma) - K\Delta]^2 - 4S(\Delta + \sigma)[S(\Delta + \sigma) - Ka_0]}}{2S(\Delta + \sigma)}$$

Then we have

$$L_{DR\text{-}G} < L_{CR} \Leftrightarrow 2S^2\Delta^2 < 2SK(\Delta + \sigma)a_0 - K^2\Delta\sigma - K\sigma\sqrt{[2S(\Delta + \sigma) - K\Delta]^2 - 4S(\Delta + \sigma)[S(\Delta + \sigma) - Ka_0]})$$

$$[2SKa_0(\Delta + \sigma) - 2S^2\Delta^2 - K^2\Delta\sigma]^2 > K^2\sigma^2[[2S(\Delta + \sigma) - K\Delta]^2 - 4S(\Delta + \sigma)[S(\Delta + \sigma) - Ka_0]]$$

If $\sigma > \frac{2S\Delta(S\Delta - Ka_0)}{K(2Sa_0 - K\Delta)}$, then

$$L_{DR\text{-}G} < L_{CR} \Leftrightarrow S\Delta^3 + K\Delta\sigma(\Delta + \sigma) > Ka_0(\Delta + \sigma)^2 \quad \Leftrightarrow \quad \frac{2S\Delta(S\Delta - Ka_0)}{K(2Sa_0 - K\Delta)} < \sigma < \frac{\Delta\sqrt{K^2\Delta^2 + 4SK\Delta(a_0 - \Delta)} - K\Delta^2 - 2K\Delta(a_0 - \Delta)}{2K(a_0 - \Delta)}$$

Otherwise (i.e., $0 < \sigma < \frac{2S\Delta(S\Delta - Ka_0)}{K(2Sa_0 - K\Delta)}$ and $\sigma > \frac{\Delta\sqrt{K^2\Delta^2 + 4SK\Delta(a_0 - \Delta)} - K\Delta^2 - 2K\Delta(a_0 - \Delta)}{2K(a_0 - \Delta)}$), $L_{DR\text{-}G} > L_{CR}$.

$$L_{DR} > L_{DR\text{-}G} \Leftrightarrow \sqrt{K^2\Delta^2 + 4SK\Delta(a_0 - \Delta)} < K\sigma + \sqrt{K^2(\Delta + \sigma)^2 + 4SK(\Delta + \sigma)(a_0 - \Delta + \alpha - \sigma)}$$

$$\Leftrightarrow 2S(a_0 - \Delta) + \sqrt{K^2\Delta^2 + 4SK\Delta(a_0 - \Delta)} > \sqrt{[2S(\Delta + \sigma) - K\Delta]^2 - 4S(\Delta + \sigma)[S(\Delta + \sigma) - Ka_0]}$$

$$\Leftrightarrow 0 < \sigma < \frac{S(a_0 - \Delta) + \sqrt{K^2\Delta^2 + 4SK\Delta(a_0 - \Delta)}}{K}$$

Otherwise (i.e., $\sigma > \dfrac{S(a_0 - \Delta) + \sqrt{K^2\Delta^2 + 4SK\Delta(a_0 - \Delta)}}{K}$), $L_{DR} < L_{DR\text{-}G}$.

J. Proof of Equation (4.4)

The structure of the differential equation (4.3)'s solution is in the form of

$$V(C) = A_1 C^{\beta_1} + A_2 C^{\beta_2} - \frac{\alpha C}{\rho - \mu} + \frac{P}{\rho} \tag{J1}$$

where β_1 and β_2 are the roots of the characteristic quadratic equation as follows:

$$\frac{1}{2}\sigma^2 \beta^2 + \left(\mu - \frac{1}{2}\sigma^2\right)\beta - \rho = 0 \tag{J2}$$

Solving (J2), we have

$$\beta_1 = \left[\frac{1}{2}\sigma^2 - \mu + \sqrt{\left(\frac{1}{2}\sigma^2 - \mu\right)^2 + 2\rho\sigma^2}\right]\bigg/ \sigma^2, \quad \beta_2 = \left[\frac{1}{2}\sigma^2 - \mu - \sqrt{\left(\frac{1}{2}\sigma^2 - \mu\right)^2 + 2\rho\sigma^2}\right]\bigg/ \sigma^2$$

Since when $c \to 0$, the maintenance cost becomes negligible (Assumption 4.4), which indicates that the value of remanufacturing option approaches zero, and hence, $A_2 = 0$. After eliminating this speculative bubble, the general solution becomes equation (4.4).

K. Proof of Proposition 4.1

By equation (4.6), we have

$$A_1 = \frac{M'(C_R) - \rho + \mu}{(\rho - \mu)\beta_1 C_R^{\beta_1 - 1}} = \frac{\alpha - \rho + \mu}{(\rho - \mu)\beta_1 C_R^{\beta_1 - 1}} \tag{K1}$$

Substitute (K1) into (4.5), we have

$$H = -[M'(C_R) + (\beta_1 - 1)(\rho - \mu)]C_R^{\beta_1} + [M(C_R) - M(C_0)]\beta_1 C_R^{\beta_1 - 1} + [M'(C_R) - \rho + \mu]C_0^{\beta_1} \tag{K2}$$

$$= (\alpha - \rho + \mu)(\beta_1 - 1)C_R^{\beta_1} - \alpha\beta_1 C_0 C_R^{\beta_1 - 1} + (\alpha - \rho + \mu)C_0^{\beta_1} = 0 \tag{K3}$$

Hence, $H(C_R = C_0) = -(\rho - \mu)\beta_1 C_0^{\beta_1} < 0$. Also, by equation (4.5) and (K2), we have

$$\frac{dH}{dC_R} = (\alpha - \rho + \mu)\beta_1(\beta_1 - 1)C_R^{\beta_1 - 1} - \alpha\beta_1(\beta_1 - 1)C_0 C_R^{\beta_1 - 2} = A_1\beta_1(\beta_1 - 1)(\rho - \mu)C_R^{\beta_1 - 2}(C_R^{\beta_1} - C_0^{\beta_1}) > 0 \tag{K4}$$

Hence, obviously, the solution of equation (K3) exists and is unique.

L. Proof of Proposition 4.2

By equation (4.5), (4.6), and (K3), we have

$$\frac{dH}{d\beta_1} = (\alpha - \rho + \mu)[C_R^{\beta_1} + (\beta_1 - 1)C_R^{\beta_1}\ln C_R + C_0^{\beta_1}\ln C_0] - \alpha C_0 C_R^{\beta_1 - 1} - \alpha\beta_1 C_0 C_R^{\beta_1 - 1}\ln C_R$$

$$= (1 + \beta_1\ln C_R)C_R^{\beta_1 - 1}[(\alpha - \rho + \mu)C_R - \alpha C_0] - (\alpha - \rho + \mu)(C_R^{\beta_1}\ln C_R - C_0^{\beta_1}\ln C_0)$$

$$= A_1(\rho - \mu)C_R^{\beta_1 - 1}\left[(1 + \beta_1\ln C_R)(C_R^{\beta_1} - C_0^{\beta_1}) - \beta_1(C_R^{\beta_1}\ln C_R - C_0^{\beta_1}\ln C_0)\right] \tag{L1}$$

Suppose $F_1(C) = (1 + \beta_1\ln C)(C^{\beta_1} - C_0^{\beta_1}) - \beta_1(C^{\beta_1}\ln C - C_0^{\beta_1}\ln C_0)$ where $C > C_0$, then we have

$$F_1(C_0) = 0 \tag{L2}$$

$$F_1'(C) = \frac{\beta_1(C^{\beta_1} - C_0^{\beta_1})}{C} > 0 \tag{L3}$$

Obviously, $F_1(C_R) > 0$, and $dH/d\beta_1 > 0$ accordingly. Hence, by (L3), we have

$$\frac{\partial C_R}{\partial \beta_1} = -\frac{dH/d\beta_1}{dH/dC_R} < 0 \tag{L4}$$

Furthermore, by equation (4.4), we have

$$\frac{\partial \beta_1}{\partial \sigma} = -\left[2\mu^2 + 2\rho\sigma^2 - \mu\sigma^2 - 2\mu\sqrt{\left(\frac{1}{2}\sigma^2 - \mu\right)^2 + 2\rho\sigma^2}\right]\Big/\sigma^3 \tag{L5}$$

Since $\left(2\mu^2 + 2\rho\sigma^2 - \mu\sigma^2\right)^2 - 4\mu^2\left[\left(\frac{1}{2}\sigma^2 - \mu\right)^2 + 2\rho\sigma^2\right] = 4\sigma^4\rho(\rho - \mu) > 0$, we have $\frac{\partial \beta_1}{\partial \sigma} < 0$. Therefore,

$$\frac{\partial C_R}{\partial \sigma} = \frac{\partial C_R}{\partial \beta_1} \cdot \frac{\partial \beta_1}{\partial \sigma} > 0 \tag{L6}$$

Also, by equation (4.5), (4.6), (L2), and (L3) we have

$$\frac{dH}{dC_0} = -\alpha\beta_1(C_R^{\beta_1 - 1} - C_0^{\beta_1 - 1}) - (\rho - \mu)\beta_1 C_0^{\beta_1 - 1} < 0 \quad \Rightarrow \quad \frac{dC_R}{dC_0} > 0 \tag{L7}$$

M. Proof of Proposition 4.3

By equation (4.9), 4.(10), and (B2), we have

$$\overline{H} = [M'(C_R) - (\rho - \mu)(1 + d'(C_R)WL)](C_R^{\beta_1} - C_0^{\beta_1}) - [M(C_R) - M(C_0)]\beta_1 C_R^{\beta_1 - 1} + [C_R + d(C_R)WL - S](\rho - \mu)\beta_1 C_R^{\beta_1 - 1} \tag{M1}$$

$$= -(\alpha - \rho + \mu)[(\beta_1 - 1)C_R^{\beta_1} + C_0^{\beta_1}] - (\rho - \mu)d'(C_R)WL(C_R^{\beta_1} - C_0^{\beta_1}) + [\alpha C_0 + [d(C_R)WL - S](\rho - \mu)]\beta_1 C_R^{\beta_1 - 1} = 0 \tag{M2}$$

$$\frac{d\overline{H}}{dC_R} = (\rho - \mu)(C_R^{\beta_1} - C_0^{\beta_1})[WLd''(C_R) + A_1\beta_1(\beta_1 - 1)C_R^{\beta_1 - 2}] = (\rho - \mu)(C_R^{\beta_1} - C_0^{\beta_1})A_1\beta_1(\beta_1 - 1)C_R^{\beta_1 - 2} > 0 \tag{M3}$$

$$\frac{d\overline{H}}{dS} = (\rho - \mu)\beta_1 C_R^{\beta_1 - 1} > 0 \quad \Rightarrow \quad \frac{dC_R}{dS} = -\frac{d\overline{H}/dS}{d\overline{H}/dC_R} < 0 \tag{M4}$$

$$\frac{d\overline{H}}{dL} = (\rho - \mu)W[d'(C_R)(C_R^{\beta_1} - C_0^{\beta_1}) - d(C_R)\beta_1 C_R^{\beta_1 - 1}] = (\rho - \mu)W[\gamma(C_R^{\beta_1} - C_0^{\beta_1}) - (\gamma C_R - \gamma C_0 + d_0)\beta_1 C_R^{\beta_1 - 1}] \tag{M5}$$

Suppose $F_3(C) = \gamma(C^{\beta_1} - C_0^{\beta_1}) - (\gamma C - \gamma C_0 + d_0)\beta_1 C^{\beta_1 - 1}$ where $C > C_0$, then we have

$$F_3(C_0) = -d_0\beta_1 C_0^{\beta_1-1} < 0 \tag{M6}$$

$$F_3{}'(C) = -\beta_1(\beta_1-1)C^{\beta_1-2}(C-C_0) < 0 \tag{M7}$$

Therefore, $F_3(C) < 0$ and $dH/dL < 0$ accordingly, which indicates that

$$\frac{dC_R}{dL} = -\frac{d\bar{H}/dL}{d\bar{H}/dC_R} > 0 \tag{M8}$$

N. Proof of Proposition 4.4

$$\frac{dQ}{dS} = \frac{dQ}{dC_R} \cdot \frac{dC_R}{dS} = \frac{d\left[\dfrac{W(\mu-\sigma^2/2)d(C_R)}{\ln C_R - \ln C_0}\right]}{dC_R} \cdot \frac{dC_R}{dS} = W(\mu-\frac{\sigma^2}{2}) \cdot \frac{C_R d'(C_R)(\ln C_R - \ln C_0) - d(C_R)}{C_R(\ln C_R - \ln C_0)^2} \cdot \frac{dC_R}{dS} \tag{N1}$$

$$= W(\mu - \frac{\sigma^2}{2}) \cdot \frac{\gamma C_R(\ln C_R - \ln C_0) - \gamma(C_R - C_0) - d_0}{C_R(\ln C_R - \ln C_0)^2} \cdot \frac{dC_R}{dS} \tag{N2}$$

Suppose $F_4(C) = \gamma C(\ln C - \ln C_0) - \gamma(C - C_0) - d_0$ where $C > C_0$, then we have $dF_4(C)/dC = \ln C - \ln C_0 > 0$ and

$F_4(C_0) = -d_0 < 0$. Hence, there exists $\bar{C}$ that equation $F_4(\bar{C}) = 0$. Hence, by (M2), (M4), and (M2), we have

$$\frac{dQ}{dS} = \frac{dQ}{dC_R} \cdot \frac{dC_R}{dS} > 0 \iff \frac{dQ}{dC_R} < 0 \iff C_R < \bar{C} \iff$$

$$S > WLd(\bar{C}) + \frac{\alpha C_0}{\rho - \mu} - \frac{(\alpha - \rho + \mu)[(\beta_1-1)\bar{C}^{\beta_1} + C_0^{\beta_1}] + (\rho-\mu)WL(\bar{C}^{\beta_1} - C_0^{\beta_1})d'(\bar{C})}{(\rho-\mu)\beta_1\bar{C}^{\beta_1-1}}$$

$$= WL[\gamma(\bar{C} - C_0) + d_0] + \frac{\alpha C_0}{\rho - \mu} - \frac{(\alpha - \rho + \mu)[(\beta_1-1)\bar{C}^{\beta_1} + C_0^{\beta_1}] + (\rho-\mu)\gamma WL(\bar{C}^{\beta_1} - C_0^{\beta_1})}{(\rho-\mu)\beta_1\bar{C}^{\beta_1-1}} \tag{N3}$$

O. Proof of Proposition 4.5

Similarly to the deduction process in Appendix E, by (M2), (M8), and (M2), we have

$$\frac{dQ}{dL} > 0 \iff \frac{dQ}{dC_R} > 0 \iff C_R > \bar{C} \iff L > \frac{(\alpha - \rho + \mu)[(\beta_1-1)\bar{C}^{\beta_1} + C_0^{\beta_1}] - [\alpha C_0 - S(\rho-\mu)]\beta_1\bar{C}^{\beta_1-1}}{W(\rho-\mu)[\beta_1\bar{C}^{\beta_1-1}d(\bar{C}) - (\bar{C}^{\beta_1} - C_0^{\beta_1})d'(\bar{C})]}$$

$$\iff L > \frac{(\alpha - \rho + \mu)[(\beta_1-1)\bar{C}^{\beta_1} + C_0^{\beta_1}] - [\alpha C_0 - S(\rho-\mu)]\beta_1\bar{C}^{\beta_1-1}}{W(\rho-\mu)\left[\beta_1\bar{C}^{\beta_1-1}[\gamma(\bar{C}-C_0) + d_0] - \gamma(\bar{C}^{\beta_1} - C_0^{\beta_1})\right]} \tag{O1}$$

P. Proof of Proposition 4.6

By Proposition 4.5, we have

$$S^* = WL[\gamma(\bar{C} - C_0) + d_0] + \bar{C} + \frac{\alpha(C_0 - \bar{C})}{\rho - \mu} + \frac{[\alpha - (\rho-\mu)(1 + \gamma WL)](\bar{C}^{\beta_1} - C_0^{\beta_1})}{(\rho-\mu)\beta_1\bar{C}^{\beta_1-1}} \tag{P1}$$

Let $K(\beta_1) = \dfrac{\bar{C}^{\beta_1} - C_0^{\beta_1}}{\beta_1\bar{C}^{\beta_1-1}}$, we have

$$\frac{dK}{d\beta_1} = \frac{(\overline{C}^{\beta_1}\ln\overline{C} - C_0^{\beta_1}\ln C_0)\beta_1\overline{C}^{\beta_1-1} - (\overline{C}^{\beta_1} - C_0^{\beta_1})(\overline{C}^{\beta_1-1} + \beta_1\overline{C}^{\beta_1-1}\ln\overline{C})}{(\beta_1\overline{C}^{\beta_1-1})^2} = -\frac{\overline{C}^{\beta_1-1}[(\overline{C}^{\beta_1} - C_0^{\beta_1}) - \beta_1 C_0^{\beta_1}(\ln\overline{C} - \ln C_0)]}{(\beta_1\overline{C}^{\beta_1-1})^2} \quad \text{(P2)}$$

Let $N(C) = (C^{\beta_1} - C_0^{\beta_1}) - \beta_1 C_0^{\beta_1}(\ln C - \ln C_0)$, then ⬚⬚ and $\frac{dN}{dC} = \beta_1 C^{\beta_1-1} - \frac{\beta_1 C_0^{\beta_1}}{C} = \frac{\beta_1(C^{\beta_1} - C_0^{\beta_1})}{C} > 0$.

Hence,

$$N(\overline{C}) = (\overline{C}^{\beta_1} - C_0^{\beta_1}) - \beta_1 C_0^{\beta_1}(\ln\overline{C} - \ln C_0) > 0 \quad \text{(P3)}$$

Therefore, $\frac{dK}{d\beta_1} < 0$, which results in $\frac{dS^*}{d\beta_1} < 0$. Furthermore, since $\frac{d\beta_1}{d\sigma} < 0$, we have $\frac{dS^*}{d\sigma} < 0$.

Also, by Proposition 4.5, we have

$$L^* = \frac{(\alpha - \rho + \mu)[(\beta_1 - 1)\overline{C}^{\beta_1} + C_0^{\beta_1}] - [\alpha C_0 - S(\rho - \mu)]\beta_1\overline{C}^{\beta_1-1}}{\gamma W(\rho - \mu)[\beta_1\overline{C}^{\beta_1}(\ln\overline{C} - \ln C_0) - (\overline{C}^{\beta_1} - C_0^{\beta_1})]} \quad \text{(P4)}$$

$$\frac{dL^*}{d\beta_1} = \frac{(\alpha - \rho + \mu)(\ln\overline{C} - \ln C_0)\overline{C}^{\beta_1}[\beta_1 C_0^{\beta_1} + \overline{C}^{\beta_1} - C_0^{\beta_1} - \beta_1 C_0^{\beta_1}(\ln\overline{C} - \ln C_0)] + \alpha\overline{C}^{\beta_1-1}C_0[\overline{C}^{\beta_1} - C_0^{\beta_1} - \beta_1 C_0^{\beta_1}(\ln\overline{C} - \ln C_0)]}{\gamma^2 W^2(\rho - \mu)^2[\beta_1\overline{C}^{\beta_1}(\ln\overline{C} - \ln C_0) - (\overline{C}^{\beta_1} - C_0^{\beta_1})]^2}$$

$$\text{(P5)}$$

By (P3), we have $\overline{C}^{\beta_1} - C_0^{\beta_1} - \beta_1 C_0^{\beta_1}(\ln\overline{C} - \ln C_0) > 0$. Hence, $\frac{dL^*}{d\beta_1} > 0$ and $\frac{dL^*}{d\sigma} < 0$.

Q. Proof of Equation (5.6)

The structure of the differential equation (5.5)'s solution is in the form of

$$V(C_D, C_N) = A_D C_D^{\beta_D} + \overline{A}_D C_D^{\overline{\beta}_D} + A_N C_N^{\beta_N} + \overline{A}_N C_N^{\overline{\beta}_N} - \frac{C_D}{\rho - \mu_D} - \frac{C_N}{\rho - \mu_N} + \frac{P}{\rho} \quad \text{(Q1)}$$

where β_D and β_N are the roots of the characteristic quadratic equations (Q2) and (Q3), respectively.

$$\frac{1}{2}\sigma_D^2\beta_D^2 + (\mu_D - \frac{1}{2}\sigma_D^2)\beta_D - \rho = 0 \quad \text{(Q2)}$$

$$\frac{1}{2}\sigma_N^2\beta_N^2 + (\mu_N - \frac{1}{2}\sigma_N^2)\beta_N - \rho = 0 \quad \text{(Q3)}$$

Solving (Q2) and (Q3), we have

$$\beta_D = \left[\frac{1}{2}\sigma_D^2 - \mu_D + \sqrt{\left(\frac{1}{2}\sigma_D^2 - \mu_D\right)^2 + 2\rho\sigma_D^2}\right]\bigg/\sigma_D^2 > 1, \quad \overline{\beta}_D = \left[\frac{1}{2}\sigma_D^2 - \mu_D + \sqrt{\left(\frac{1}{2}\sigma_D^2 - \mu_D\right)^2 + 2\rho\sigma_D^2}\right]\bigg/\sigma_D^2 < 0$$

$$\beta_N = \left[\frac{1}{2}\sigma_N^2 - \mu_N + \sqrt{\left(\frac{1}{2}\sigma_N^2 - \mu_N\right)^2 + 2\rho\sigma_N^2}\right]\bigg/\sigma_N^2 > 1, \quad \overline{\beta}_N = \left[\frac{1}{2}\sigma_N^2 - \mu_N - \sqrt{\left(\frac{1}{2}\sigma_N^2 - \mu_N\right)^2 + 2\rho\sigma_N^2}\right]\bigg/\sigma_N^2 < 0$$

When c_N is finite and $c_D \to 0$, the product value is finite; when c_D is finite and $c_N \to 0$, the product

value is also finite. Therefore, $\bar{A}_D = \bar{A}_N = 0$ and the general solution becomes equation (5.6).

R. Proof of Proposition 5.2

By equation (5.6), we have

$$\frac{\partial \beta_N}{\partial \sigma_N} = -\left[2\mu_N^2 + 2\rho\sigma_N^2 - \mu_N\sigma_N^2 - 2\mu_N\sqrt{\left(\frac{1}{2}\sigma_N^2 - \mu_N\right)^2 + 2\rho\sigma_N^2} \right] \Bigg/ \sigma_N^3 < 0 \tag{R1}$$

$$\frac{\partial \beta_N}{\partial \rho} = \frac{2\rho}{\sigma_N\sqrt{\left(\frac{1}{2}\sigma_N^2 - \mu_N\right)^2 + 2\rho\sigma_N^2}} > 0 \tag{R2}$$

Since $(2\mu_N^2 + 2\rho\sigma_N^2 - \mu_N\sigma_N^2)^2 - 4\mu_N^2\left[\left(\frac{1}{2}\sigma_N^2 - \mu_N\right)^2 + 2\rho\sigma_N^2\right] = 4\sigma_N^2\rho(\rho - \mu_N) > 0$, we have $\frac{\partial \beta_N}{\partial \sigma_N} < 0$. Hence,

$\frac{\partial C_{NR}}{\partial \sigma_N} = \frac{\partial C_{NR}}{\partial \beta_N}\frac{\partial \beta_N}{\partial \sigma_N} > 0$. Also, by Proposition 1, we have $\frac{\partial C_{NR}}{\partial \beta_N} = -\frac{K_R(\rho - \mu_N)}{(\beta_N - 1)^2} < 0$, $\frac{\partial C_{NR}}{\partial K_R} = \frac{(\rho - \mu_N)\beta_N}{\beta_N - 1} > 0$, and

$\frac{\partial C_{NR}}{\partial \rho} = \frac{K_R[\beta_N + (\rho - \mu_N)(\partial \beta_N / \partial \rho)]}{\beta_N - 1} > 0$.

S. Proof of Proposition 5.3

Substituting the smooth-pasting conditions (5.11) and (5.12) into the value-matching (5.10), we have

$$\frac{C_{DA}}{(\rho - \mu_D)\beta_D} + \frac{C_{NA}}{(\rho - \mu_N)\beta_N} - \frac{C_{DA}}{\rho - \mu_D} - \frac{C_{NA}}{\rho - \mu_N} + K_A = 0 \tag{S1}$$

$$\Rightarrow \quad F(C_{DA}, C_{NA}) = (\rho - \mu_N)(\beta_D - 1)\beta_N C_{DA} + (\rho - \mu_D)\beta_D(\beta_N - 1)C_{NA} - (\rho - \mu_D)(\rho - \mu_N)\beta_D\beta_N K_A = 0 \tag{S2}$$

Therefore, when $F(C_D, C_N) \geq 0$, the difference between the new project value after the abandonment

and the current project value (before the abandonment) is not less than the manufacturing cost of the

new product, which implies the exercise of the abandonment option.

T. Proof of Proposition 5.4 and Corollary 5.1

Substituting the smooth-pasting conditions (5.11) and (5.12) into the value-matching (5.10), we have

$$F(C_{DA}, C_{NA}) - K_A = \frac{C_{DA} - C_{D0}}{\rho - \mu_D} + \frac{C_{NA} - C_{N0}}{\rho - \mu_N} - \frac{C_{DA}^{\beta_D} - C_{D0}^{\beta_D}}{(\rho - \mu_D)\beta_D C_{DA}^{\beta_D - 1}} - \frac{C_{NA}^{\beta_N} - C_{N0}^{\beta_N}}{(\rho - \mu_N)\beta_N C_{NA}^{\beta_N - 1}} - K_A = 0 \tag{T1}$$

Therefore, when $F(C_D, C_N) \geq 0$, the difference between the new project value after the replacement and the current project value (before the replacement) is not less than the manufacturing cost of the new product, which implies the exercise of the replacement option. Furthermore,

$$\frac{\partial F(C_D, C_N)}{\partial C_D} = \frac{(\beta_D - 1)(C_D^{\beta_D} - C_{D0}^{\beta_D})}{(\rho - \mu_D)\beta_D C_D^{\beta_D}} > 0 \tag{T2}$$

$$\frac{\partial F(C_D, C_N)}{\partial C_N} = \frac{(\beta_N - 1)(C_N^{\beta_N} - C_{N0}^{\beta_N})}{(\rho - \mu_N)\beta_N C_N^{\beta_N}} > 0 \tag{T3}$$

U. Proof of Proposition 5.5

By Proposition 3, given that the non-durable-part O&M cost is fixed, we have

$$\frac{\partial C_D}{\partial \beta_D} = -\frac{\partial [F(C_D, C_N) - K_A]/\partial \beta_D}{\partial [F(C_D, C_N) - K_A]/\partial C_D} = \frac{C_D[\beta_D C_{D0}^{\beta_D}(\ln C_D - \ln C_{D0}) - (C_D^{\beta_D} - C_{D0}^{\beta_D})]}{\beta_D(\beta_D - 1)(C_D^{\beta_D} - C_{D0}^{\beta_D})} \tag{U1}$$

$$\frac{\partial C_D}{\partial \beta_N} = -\frac{\partial [F(C_D, C_N) - K_A]/\partial \beta_N}{\partial [F(C_D, C_N) - K_A]/\partial C_D} = \frac{(\rho - \mu_D)\beta_D C_D^{\beta_D}[\beta_N C_{N0}^{\beta_N}(\ln C_N - \ln C_{N0}) - (C_N^{\beta_N} - C_{N0}^{\beta_N})]}{(\rho - \mu_N)(\beta_D - 1)\beta_N^2 C_N^{\beta_N - 1}(C_D^{\beta_D} - C_{D0}^{\beta_D})} \tag{U2}$$

$$\frac{\partial C_D}{\partial K_A} = -\frac{\partial [F(C_D, C_N) - K_A]/\partial K_A}{\partial [F(C_D, C_N) - K_A]/\partial C_D} = \frac{(\rho - \mu_N)\beta_D C_D^{\beta_D}}{(\beta_D - 1)(C_D^{\beta_D} - C_{D0}^{\beta_D})} \tag{U3}$$

Suppose $H(C) = \beta_D C_{D0}^{\beta_D}(\ln C - \ln C_{D0}) - (C^{\beta_D} - C_{D0}^{\beta_D})$, then $H(C_{D0}) = 0$ and $\frac{\partial H(C)}{\partial C} = -\frac{\beta_D(C^{\beta_D} - C_{D0}^{\beta_D})}{C} < 0$. Therefore, $\frac{\partial C_D}{\partial \beta_D} < 0$ and $\frac{\partial C_D}{\partial \sigma_D} = \frac{\partial C_D}{\partial \beta_D}\frac{\partial \beta_D}{\partial \sigma_D} > 0$. Similarly, $\frac{\partial C_D}{\partial \beta_N} < 0$ and $\frac{\partial C_D}{\partial \sigma_D} = \frac{\partial C_D}{\partial \beta_D}\frac{\partial \beta_D}{\partial \sigma_D} > 0$.

By Proposition 5.3, given that the durable-part O&M cost is fixed, we have

$$\frac{\partial C_N}{\partial \beta_D} = -\frac{\partial [F(C_D, C_N) - K_A]/\partial \beta_D}{\partial [F(C_D, C_N) - K_A]/\partial C_N} = \frac{C_D[\beta_D C_{D0}^{\beta_D}(\ln C_D - \ln C_{D0}) - (C_D^{\beta_D} - C_{D0}^{\beta_D})]}{\beta_N(\beta_N - 1)(C_N^{\beta_N} - C_{N0}^{\beta_N})} \tag{U4}$$

$$\frac{\partial C_N}{\partial \beta_N} = -\frac{\partial [F(C_D, C_N) - K_A]/\partial \beta_N}{\partial [F(C_D, C_N) - K_A]/\partial C_N} = \frac{(\rho - \mu_D)\beta_D C_D^{\beta_D}[\beta_N C_{N0}^{\beta_N}(\ln C_N - \ln C_{N0}) - (C_N^{\beta_N} - C_{N0}^{\beta_N})]}{(\rho - \mu_D)(\beta_N - 1)\beta_D^2 C_D^{\beta_D - 1}(C_N^{\beta_N} - C_{N0}^{\beta_N})} \tag{U5}$$

$$\frac{\partial C_N}{\partial K_A} = -\frac{\partial [F(C_D, C_N) - K_A]/\partial K_A}{\partial [F(C_D, C_N) - K_A]/\partial C_N} = \frac{(\rho - \mu_N)\beta_D C_D^{\beta_D}}{(\beta_N - 1)(C_N^{\beta_N} - C_{N0}^{\beta_N})} \tag{U6}$$

Therefore, $\frac{\partial C_N}{\partial \beta_D} < 0$ and $\frac{\partial C_N}{\partial \sigma_D} = \frac{\partial C_N}{\partial \beta_D}\frac{\partial \beta_D}{\partial \sigma_D} > 0$. Similarly, $\frac{\partial C_N}{\partial \beta_N} < 0$ and $\frac{\partial C_N}{\partial \sigma_D} = \frac{\partial C_N}{\partial \beta_D}\frac{\partial \beta_D}{\partial \sigma_D} > 0$.

V. Proof of Lemma 5.1 and Proposition 5.6

By Proposition 5.1, the maximum non-durable-part O&M cost C_{NR} is the solution of

$$(\beta_N - 1)C_{NR}^{\beta_N} - [K_R(\rho - \mu_N) + C_{N0}]\beta_N C_{NR}^{\beta_N - 1} + C_{N0}^{\beta_N} = 0 \tag{V1}$$

Substituting (V1) into $F(C_D, C_N) = K_A$ in Proposition 4, it can be derived that the minimum lower bound

of durable-part O&M cost to exercise the replacement option C_{DA} is the solution of (V2)

$$(\beta_D - 1)C_{DA}^{\beta_D} - [(K_R - K_A)(\rho - \mu_D) + C_{D0}]\beta_D C_{DA}^{\beta_D - 1} + C_{D0}^{\beta_D} = 0 \tag{V1}$$

W. Proof of Corollary 5.3

According to Proposition 5.6, suppose $G = (\beta_D - 1)\bar{C}_{DA}^{\beta_D} - [(K_R - K_A)(\rho - \mu_D) + C_{D0}]\beta_D \bar{C}_{DA}^{\beta_D - 1} + C_{D0}^{\beta_D} = 0$, we have

$$\frac{d\bar{C}_{DA}}{dK_R} = -\frac{\partial G / \partial K_R}{\partial G / \partial \bar{C}_{DA}} = \frac{-(\rho - \mu_D) + C_{D0}]\bar{C}_{DA}}{(\beta_D - 1)[\bar{C}_{DA} - C_{D0} - [(K_A - K_R)(\rho - \mu_D)]} < 0 \tag{W1}$$

REFERENCES

[1] J. Ginsburg, "Manufacturing: Once is not enough," *Business Week*, (April 16) B128-B129, 2011.

[2] http://www.xerox.com/corporate-citizenship/2011/sustainability/preservation.html, accessed on Dec. 19, 2011.

[3] N. Ludlum, B. Pritchard, T. Patten, and G. Voorhees, "Panasonic, Techneglas and Envirocycle Achieve Breakthrough in Television Picture Tube Recycling," http://www.panasonic.com/environmental/docs/crt_recycled_content_061702.pdf, last accessed on Nov. 25, 2011.

[4] V. D. R. Guide, "Production planning and control for remanufacturing: industrial practice and research needs," *Journal of Operations Management*, vol. 18, pp. 467-483, 2000.

[5] R. Geyer, L. N. Van Wassenhove, and A. Atasu, "The economics of remanufacturing under limited component durability and finite product life cycles," *Management Science*, vol. 53, pp. 88-100, 2007.

[6] C. K. Huang, Y. S. Tarng, C. Y. Chiu, and A. P. Huang, "Investigation of Machine Vision Assisted Automatic Resharpening Process of Micro-Drills," *Int. J. of Materials Processing Technology*, vol. 209, pp. 5944-5954, 2009.

[7] Xerox, www.xerox.com/corporate-citizenship-2005/Environment_Health_Safety_Report_2005.pdf, last accessed on Nov. 25, 2011.

[8] S. Goyal, S. S. Upasani, and D. M. Patel, "Designing best-in-class impact –tolerant cellular phones and other portable products," *Bell Labs Technical Journal*, vol. 3, pp. 159-174, 2002.

[9] Konica, "Konica Minolta: Sustainability Report 2004," http://www.konicaminolta.com/about/environment/report/2004/index.html, last accessed on Nov. 25, 2011.

[10] FujiFilm, "The FujiFilm Environmental Report/2001 Edition," http://www.fujifilm.com/sustainability/report/pdf/index/ff_er_2001_001.pdf, last accessed on Nov. 25, 2011.

[11] C. Savaskan, S. Bhattacharya, and L. N. Van Wassenhove, "Closed-loop supply chain models with product remanufacturing," *Management Science*, vol. 50, pp. 239-252, 2004.

[12] V. D. R. Guide, R. Teunter, and L. N. Van Wassenhove, "Matching supply and demand to maximize profits from remanufacturing," *Manufacturing & Service Operations Management*. 5, 303-316, 2003.

[13] A. Atasu, L. N. Van Wassenhove, and M. Sarvary, "Efficient Take-Back Legislation," *Production and Operations Management*, vol. 18, no. 3, pp. 243-258, 2003.

[14] RecycleWorks, http://www.recycleworks.org/ewaste/ewaste_process.html, last accessed on Nov. 25, 2011.

[15] M. Ahmetoglu, T. Altan, "Tube hydroforming: state of the art and future trends," *J. of Materials Processing Technology*, vol. 98, pp. 25-33, 2000.

[16] J. S. Slechta, G. S. William, "Design Tradeoffs for Frequency Hopped VHF net radios," *Pro. MILCOM*, pp. 421-426, 1982.

[17] R. Farkash and K. Mueller, "The Evolution of Environmental Initiatives for Toner Containers at Xerox Corporation," *Proceedings of the IEEE Symp. on Electronics and the Environment*, pp. 267-272, 1995.

[18] W. Maly, "IC Design in High-Cost Nanometer-Technologies Era," DAC 2001, Las Vegas, Nevada.

[19] E. L. Porteus, "Investing in reduced setups in the EOQ model," *Manag. Sci.*, 31, pp. 998-1010, 1985.

[20] I. Karakayali, H. Emir-Farinas, and E. Akcali, "An analysis of decentralized collection and processing of end-of-life products," *Journal of Operations Management*, vol. 25, pp. 1161-1183, 2007.

[21] Y. Xu, F. Elgh, J. A. Erkoyuncu, O. Bankole, Y. Goh, W. M. Cheung, P. Baguley, Q. Wang, P. Arundachawat, E. Shehab, L. Newnes, and R. Roy, "Cost Engineering for Manufacturing: Current and Future Research," *Int. J. of Computer Integrated Manufacturing*, pp. 1-15, 2009.

[22] J. Marcinkoski, B. D. James, J. A. Kalinoski, W. Podolski, T. Benjamin, and J. Kopasz, "Manufacturing Process Assumption used in Fuel Cell System Cost Analysis," *Journal of Power Sources*, vol. 196, pp. 5282-5292, 2011.

[23] CIWMB, http://www.calrecycle.ca.gov/Laws/regulations/title14/Chap08pt2/Article2pt4.htm, last accessed on Nov. 20, 2011.

[24] Electronics Take Back Coalition, "Comparison of State E-Waste Laws," http://www.electronicstakeback.com/wp-content/uploads/Detailed%20State%20Law%20Comparison%20ALL, last accessed on Nov. 20, 2011.

[25] Kodak, "Global Sustainability: Eastman Kodak Company 2008 Annual Report," http://webstag.kodak.com/eknec/PageQuerier.jhtml?pq-path=2879/7196/2300281/2300602/6170&pq-locale=en US, last accessed on Nov. 25, 2011.

[26] A. Atasu, B. Toktay, and L. N. Van Wassenhove, "The Impact of Collection Cost Structure on Reverse Channel Choice", *Submitted to Management Science*, 2009.

[27] R. P. Dant and P. D. Berger, "Modeling cooperative advertising decisions in franchising," *The J. of the Oper. Res. Society*, vol. 47, pp. 1120-1136, 1996.

[28] L. Shih, Y. Chang, and Y. Lin, "Intelligent Evaluation Approach for Electronic Product Recycling via Case-based Reasoning," *Advanced Engineering Informatics*, vol. 20, pp. 137-145, 2005.

[29] I. Jeong and K. Lee, "Assessment of the Ecodesign Improvement Options using the Global Warming and Economic Performance Indicators," *Journal of Cleaner Production*, vol. 17, pp. 1206-1213, 2009.

[30] R. Wojanowski, V. Verter, and T. Boyaci, "Retail Collection Network Design under Deposit Refund", *Computers and Operations Research*, vol. 34, no. 2, pp. 324-345, 2007.

[31] J. Sheehan and J. Sheil, "Investigating the Relationship between Design-for-Reliability/Design-for-the- Environment Investment, and Field Warranty and End-of-Life Costs," *Quality Tech. & Quant. Mgnt.*, vol. 4, no. 2, pp. 251-263, 2007.

[32] R. W. Parks, "The demand and supply of durable goods and durability," *The Amer. Econ. Rev.*, vol. 64, pp. 37-55, 1974.

[33] D. Hammond and P. Beullens, "Closed-loop supply chain network equilibrium under legislation," *European J. of Oper. Res*., vol. 183, pp. 895-908, 2007.

[34] EPA, Electronic Waste Management in the United States - *Approach 1*, http://www.epa.gov/osw/conserve/materials/ecycling/docs/app-1.pdf, last accessed on Nov. 25, 2011.

[35] Council of State Government, Electronic Waste Recycling Act California, 2009.

[36] Consumer Electronic Product Recycling - State and Federal Legislative Initiatives. 2007.

[37] Revenue Neutral Carbon Tax. http://www.livesmartbc.ca/government/carbon_tax.html, last accessed on Nov. 20, 2011.

[38] K. Palmer and M. Walls, "Extended Product Responsibility: An economic Assessment of Alternative Policies," *Resources for the Future Discussion*, pp. 99-12, 1999.

[39] Mrozek, "Revenue Neutral Deposit/Refund Systems," *Environ. Resource Econ*, 17, 183- 193, 2000.

[40] T. W. McGuire and R. Staelin, "An industry equilibrium analysis of downstream vertical integration," *Marketing Science*, vol. 2, pp. 161-191, 1983.

[41] T. Mikaelian, D. J. Nightingale, D. H. Rhodes, and D. E. Hastings, "Real options in enterprise architecture: a holistic mapping of mechanisms and types for uncertainty management," *IEEE Transactions on Engineering Management*, vol. 58, pp. 457-470, 2011.

[42] S. Majd and R. S. Pindyck, "Time to build, option value, and investment decisions," *Journal of Financial Economics*, vol. 18(1), pp. 7-27, 1987.

[43] R. T. Lund, "Remanufacturing: The experience of the United States and implications for developing countries." *World Bank,* Washington D.C., 2005.

[44] D. Hammond and P. Beullens, "Closed-loop supply chain network equilibrium under legislation," *European Journal of Operation Research*, vol. 183, pp. 895-908, 2007.

[45] M. I. Mazhar, S. Kara, and H. Kaebernick, "Remaining life estimation of used components in consumer products: Life cycle data analysis by Weibull and artificial neural networks," *Journal of Operations Management*, vol. 25, pp. 1184-1193, 2007.

[46] P. L. Swan, "Durability of Consumption Goods," *The American Economic Review*, 60, 884-894, 1970.

[47] T. Cooper, "Slow consumption: reflections on product life spans and the throwaway society," *Journal of Industrial Ecology*, vol. 9, pp. 51-67, 2005.

[48] R. R. Marathe and S. M. Ryan, "Capacity expansion under a service-level constraint for uncertain demand with lead times," *Naval Research Logistics*, vol. 56, 250-263, 2009.

[49] G. De Sensi, F. Longo, and G. Mirabelli, "Inventory policies analysis under demand patterns and lead times constraints in a real supply chain," *International Journal of Production Research*, vol. 46, pp. 6997-7016, 2008.

[50] M. I. Mazhar, S. Kara, and H. Kaebernick, "Remaining life estimation of used components in consumer products: Life cycle data analysis by Weibull and artificial neural networks," *Journal of Operations Management*, vol. 25, pp. 1184-1193, 2007.

[51] Q. Weninger and R. E. Just, "Firm dynamics with tradable output permits," *American Journal of Agricultural Economics*, vol. 84, pp. 572-584, 2002.

[52] L. Zhu, "A simulation based real options approach for the investment evaluation of nuclear power," *Computers & Industrial Engineering*, vol. 63, pp. 585-593, 2012.

[53] J. Zambujal-Oliveira and J. Duque, "Operational asset replacement strategy: A real options approach," *European Journal of Operational Research*, vol. 210, 318-325, 2011.

[54] C. Chung and H Wee, "Green-component life-cycle value on design and reverse manufacturing in semi-closed supply chain," *International Journal of Production Economics*, 113, pp. 528-545, 2008.

[55] R. A. Meyer, "Equipment replacement under uncertainty," *Management Science*, 17, 750-758, 1971.

[56] R. H. Yeh and M. Y. Chen, "Optimal preventive-maintenance warranty policies for repairable prod-ucts with age-dependent maintenance cost," *International Journal of Reliability, Quality and Safety Engineering*, vo. 12, pp. 111-125, 2005.

[57] M. Galbreth and J. Blackburn, "Optimal acquisition quantities in remanufacturing with condition uncertainty", *Production and Operations Management*, vol. 19, pp. 61-69, 2010.

[58] M. Denizel, M. Ferguson, and G. C. Souza, "Multiperiod Remanufacturing Planning with Uncertain Quality of Inputs," *IEEE Transactions on Engineering Management*, vol. 57, pp. 394-404, 2010.

[59] A. K. Dixit and R. S. Pindyck, "Investment under uncertainty," Princeton University Press, Princeton, NJ, 1994.

[60] H. Evan, A. Schrecongost, B. Bailey, and A. Morris, "The prospects for landfill gas-to-energy projects in West Virginia," Morgantown, *The Mountain Institute and Downstream Strategies*, Morgantown, WV, May, 2006.

[61] W. Kerr and C. Ryan, "Eco-efficiency gains from remanufacturing: a case study of photocopier re-manufacturing at Fuji Xerox Australia," *Journal of Cleaner Production*, vol. 9, pp. 75-81, 2001.

[62] K. Inderfurth, A. G. de Kok, and S. D. P. Flapper, "Product recovery in stochastic remanufacturing systems with multiple reuse options," *European Journal of Operations Research*, 133, 130-152, 2001.

[63] Bousquin J., Gambeta, E., Esterman, M., and Rothenberg, S., "Life cycle analysis in the printing in-dustry: a critical review," *Journal of Industrial Ecology*, vol. 16, pp. 195-205 ,2012.

[64] Bhattacharya, S., Guide V. D. R., and Van Wassenhove, L. N. Optimal order quantities with remanu-facturing across new product generations. *Prod. and Operations Management*, 15, 421-431, 2006.

[65] Ayres, R. U. and Ayres L. W., *A handbook of industrial ecology*. Edward Elgar, Northampton, MA, U.S.A., 2002

[66] Goldwasser, S. M., Printer and photocopier troubleshooting and repair collection. Available at http://repairfaq.ece.drexel.edu/sam/printfaq.htm (accessed on June 19, 2013).

[67] Weninger, Q. and Just, R. E., Firm dynamics with tradable output permits. *American Journal of Agri-cultural Economics*, 84, 572-584, 2002.

[68] Zhu, L., A simulation based real options approach for the investment evaluation of nuclear power. *Computers & Industrial Engineering*, 63, 585-593, 2012.

[69] Joo, S., Scheduling preventive maintenance for modular designed components: a dynamic ap-proach. *European Journal of Operational Research*, 192, 512-520, 2007.

[70] R. Subramanian, S. Gupta, and B. Talbot, "Product design and supply chain coordination under ex-tended producer responsibility," *Production and Operations Management*, 18(3), pp. 259-277, 2009.

[71] B. Toktay, L. Wein, and S. Zenios, "Inventory management of remanufacturable products," *Man-agement Science*, vol. 46, pp. 1412-1426, 2000.

[72] Van der Laan, and E. Salomon, "Production planning and inventory control with remanufacturing and disposal," *European J. of Operations Res*, vol. 102, pp. 264-278, 1997.

[73] M. Fleischmann, P. Beullens, J. Bloemhof-Ruwaard, and L. N. Van Wassenhove, "The impact of product recovery on logistics network design," *Prod. Operations Management*, 10, 156-173, 2001.

[74] G. Ferrer and C. Whybark, "Material planning for a remanufacturing facility," *Prod. Oper. Manag*, vol. 10, pp. 112-124, 2001.

[75] L. G. Debo, L. B. Toktay, and L. N. Van Wassenhove, "Market segmentation and product technology selection for remanufacturing products," *Management Science*, vol. 51, pp. 1193-1245, 2005.

[76] M. Kleiner, M. Geiger, and A. Klaus, *CIRP Annals - Manufacturing Technology*. 52, pp. 21-542, 2003.

[77] D. Galiana, A. Motto, and F. Bouffard, "Reconciling Social Welfare, Agent Profits, and Consumer Payments in Electricity Pools," *IEEE Transactions On Power Systems*, vol. 18, pp. 452-459, 2003.

[78] C. Carraro and G. Topa, "Taxation and environmental innovation. Control and Game-Theoretic Models of the Environment," *Annals of the International Society of Dynamic Games*. vol. 2, 1995.

[79] A. Robotics, T. Boyaci, and V. Verter, "Investing in reusability of products of uncertain remanufac-
turing cost: The role of inspection capabilities," *International Journal of Production Economics*, vol.
140, pp. 385-395, 2012.

[80] A. S. Siddiqui and C. Marnay, "Distributed generation investment by a microgrid under uncertainty,"
Energy, vol. 33(12), pp. 1729-1737, 2008.

[81] R. Adkins and D. Paxson, "Reciprocal energy-switching options," *The Journal of Energy Markets*. vol.
4, pp. 91-120, 2011.

[82]M. Wickart and R. Madlener, "Optimal technology choice and investment timing: a stochastic model
of industrial cogeneration vs. heat-only production," *Energy Economics*, vol. 29, pp. 934-952, 2007.

[83] M. Ye, "Optimal replacement policy with stochastic maintenance and operation costs," *European
Journal of Operational Research*, vol. 44, pp. 84-94, 1990.

[84] Min, J. K., Lou, C. and Wang, C. An exit and entry study of renewable power producers: a real op-
tions approach. *The Engineering Economist*, 57, 55-75, 2012.

[85] T. Mikaelian, D. J. Nightingale, D. H. Rhodes, and D. E. Hastings, "Real options in enterprise architec-
ture: a holistic mapping of mechanisms and types for uncertainty management," *IEEE Transactions
on Engineering Management*, vol. 58, pp. 457-470, 2011.

[86] M. A. Cardin, G. L. Kolfschoten, D. D. Frey, R. Neufville, O. L. Weck, and D. M. Geltner, "Empirical
evaluation of procedures to generate flexibility in engineering systems and improve lifecycle perfor-
mance," *Research in Engineering Design*, pp. 1-19, 2012.

[87] H. B. Nembhard and M. Aktan, Real Options in Engineering Design, Operations, and Management.
Boca Raton, FL, Untied States: Taylor and Francis, 2010.

[88] R. de Neufville and S. Scholtes, Flexibility in Engineering Design. Cambridge, MA, U.S.A: MIT Press,
2011.

[89] B. Efron and R. Tibshirani, "Bootstrap Methods for Standard Errors, Confidence Intervals, and Other

Measures of Statistical Accuracy," *Statistical Science*, vol. 1(1), pp. 54-75, 1986.